Efficient Modified Estimators for the Population Mean Using Auxiliary Variables

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Abstract

This paper presents three modified estimators to estimate the population mean by utilizing information on auxiliary variables under simple random sampling without replacement (SRSWOR). Motivation for the proposed estimators is based on Yadav *et al.* (2019). Furthermore, the expressions for bias and mean squared error (MSE) of proposed estimators were obtained and compared with other relevant existing estimators through theoretical and empirical studies. It is shown that the proposed estimators perform better than other estimators.

Keywords: Modified estimators, Auxiliary variable, Bias, MSE

Introduction

In the sampling theory literature, many authors have recommended using auxiliary information for increasing the efficiency of their estimators for estimating the population parameters. The aim of these recommendations is to use efficient estimators for making inferences about the unknown population parameters such as population total, population mean, population proportion, or population variance. One of the population parameters that has been widely studied and used, is the population mean. When the correlation between auxiliary and study variables is positive, the ratio estimator of Cochran (1940) is considered to be the most practicable. On the other hand, if the correlation between auxiliary and study variables is negative, the product estimator of Robson (1957), which is rediscovered by Murthy (1964), is employed quite effectively.

Many authors have applied the Cochran (1940) estimator for population mean using known parameters of auxiliary variables. For instance, Sisodia and Dwivedi (1981) have used the coefficient of variation (C_x) of the auxiliary variable for proposing a modified ratio estimator and they showed that their modified estimator was more efficient than Cochran (1940) estimator in some cases. Motivated by Ray and Singh (1981), Kadilar and Cingi (2004) developed traditional and other ratio-type estimators in simple random sampling for mean estimation. By applying the estimator in Upadhyaya and Singh (1999), Kadilar and Cingi (2006) also suggested the ratio estimators for estimating the population mean using the information on the C_x and the coefficient of kurtosis (β_2) of the auxiliary variable.

Al-Omari, Jemain, and Ibrahim (2009) proposed modified ratio estimators of the mean estimation using simple random sampling (SRS) and ranked set sampling (RSS) when the first or third quartiles of the auxiliary variable are available. Nonetheless, Singh, Upadhyaya, and Tailor (2009) established ratio-cum-product type exponential estimator for the population mean of the study variable using two auxiliary information. Yan and Tian (2010) used of skewness coefficient (β_1) of an auxiliary variable for improvement of some ratio-type estimators in estimating the population mean by adapting Kadilar and Cingi (2004) estimator.

Recently, Singh et al. (2012) proposed the estimation of finite population mean in two-phase sampling with the known coefficient of variation of an auxiliary character. Subramani and Kumarpandiyan (2012) worked out a class of modified ratio estimators for estimation of population mean of the study variable using the linear combination of the known values of the coefficient of variation and the median of the auxiliary variable. Jeelani, Maqbool, and Mir (2013) suggested two modified ratio estimators of population mean using the linear combination of coefficient of skewness and quartile deviation of the auxiliary variable. Later Jerajuddin and Kishun (2016) proposed new modified ratio estimators for estimating the population mean using the size of the sample, selected from the population under simple random sampling. Moreover, Singh et al. (2004) proposed a modified ratio estimator using power transformation in the estimation of population mean of the study variable. Singh and Tailor (2005) also proposed a modified ratio-cum-product estimators of finite population mean using a known correlation coefficient between two auxiliary variables.

In addition, some authors proposed to use some coefficients and develop new product estimators by adding them into the traditional ones. For example, Pandey and Dubey (1988) used known values of coefficient of variation of auxiliary variables in simple random sampling for creating a modified product estimator for mean estimation. Further, Singh (2003) suggested the modified product estimator for estimating the population mean of the study variable for negatively correlated auxiliary variables. Singh and Tailor (2003) utilized information on known correlation coefficient of auxiliary variable and suggested another product estimator of population mean under a simple random sampling scheme.

One can see that there were many authors who extended and developed ratio estimators to estimate the population mean using known auxiliary variables. Therefore, Khoshnevisan et al. (2007) proposed a general family of estimators to estimate population mean that covers the other existing ratio estimators. The estimator by Khoshnevisan et al. (2007) is given as follows:

$$t_1 = \bar{y}\left(\frac{a\bar{X}+c}{\alpha(a\bar{x}+c)+(1-\alpha)(a\bar{X}+c)}\right)^g, \tag{1}$$

where $\bar{y}$ and $\bar{x}$ are the sample means of study and auxiliary variables. $\bar{X}$ is the population mean of the auxiliary variable. The consonants $a(a \neq 0)$ and c are either real numbers or functions of the auxiliary variable such as variance $(S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i-\bar{X})^2)$, coefficient of variation $(C_x = S_x/\bar{X})$, and correlation coefficient $(\rho = S_{yx}/S_yS_x)$. The consonants α and g are real numbers to be determined.

The bias and MSE of this family of Khoshnevisan et al. (2007) estimators are respectively shown as:

$$Bias(t_1) = \frac{(1-f)}{n}\bar{Y}\alpha g\theta C_x^2\left[\frac{(g+1)}{2}\alpha\theta - C\right], \tag{2}$$

$$MSE(t_1) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \alpha g\theta C_x^2(\alpha\theta g - 2C)\right], \tag{3}$$

where $f = n/N$, $\theta = a\bar{X}/(a\bar{X}+b)$, $C_y^2 = S_y^2/\bar{Y}^2$, $S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}(y_i-\bar{Y})^2$, $C_{yx} = S_{yx}/\bar{Y}\,\bar{X}$, $S_{yx} = (N-1)^{-1}\sum_{i=1}^{N}(y_i-\bar{Y})(x_i-\bar{X})$, and $C = \rho C_y/C_x$.

However, the estimator of Khoshnevisan et al. (2007) is quite a difficult form to use in practice. Therefore, Yadav et al. (2019) improved the estimator of Khoshnevisan et al. (2007) by using the assumption of the values of α and g equal to one and adding the consonants of b and d in equation (1), as:

$$t_2 = \bar{y}\left(\frac{ab\bar{X}+cd}{ab\bar{x}+cd}\right), \tag{4}$$

The bias and MSE Yadav et al. (2019) estimators are respectively shown as:

$$Bias(t_2) = \frac{(1-f)}{n}\bar{Y}\eta C_x^2(\eta - C), \tag{5}$$

$$MSE(t_2) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \eta C_x^2(\eta - 2C)\right], \tag{6}$$

where $\eta = ab\bar{X}/(ab\bar{X}+cd)$.

In this paper the authors suggest the modification of Yadav et al. (2019) by using the concept of power transformation under the SRSWOR scheme. The expressions in terms of bias and MSE of proposed estimators were obtained. In addition, comparative studies of the proposed estimators with other relevant existing estimators have been considered through the theoretical and empirical studies, which show the efficiency of the proposed estimators was better than the other estimators.

Modified Estimators

The purpose of the authors was to create the modified estimator t_3 by adjusting the estimator of Yadav et al. (2019) when the correlation between study and auxiliary variables of this estimator is negative. The modified estimator t_3 is given as follows:

$$t_3 = \bar{y}\left(\frac{ab\bar{x}+cd}{ab\bar{X}+cd}\right), \tag{7}$$

By applying power transformation, the authors suggest replacing the constant g in both of estimators t_2 and t_3 to produce the following formula:

$$t_4 = \bar{y}\left(\frac{ab\bar{X}+cd}{ab\bar{x}+cd}\right)^g, \tag{8}$$

$$t_5 = \bar{y}\left(\frac{ab\bar{x}+cd}{ab\bar{X}+cd}\right)^g. \tag{9}$$

To obtain the bias and MSE of t_3, t_4, and t_5 under SRSWOR, let us define

$$\bar{y} = \bar{Y}(1+e_0) \text{ and } \bar{x} = \bar{X}(1+e_1) \tag{10}$$

where e_0 and e_1 are the sampling error on auxiliary and study variables, respectively.

Further, one may assume that

$$E(e_0) = E(e_1) = 0\ . \tag{11}$$

When the population parameter of the auxiliary variable is known, after solving the expectations, the following expression is obtained as

$$E(e_0^2) = \frac{(1-f)}{n}C_y^2,\quad E(e_1^2) = \frac{(1-f)}{n}C_x^2,\ \text{ and }\ E(e_0e_1) = \frac{(1-f)}{n}CC_x^2, \tag{12}$$

The bias of t_3 can be found as follows:

$$Bias(t_3) = E(t_3 - \bar{Y})$$

$$= E\left[\bar{y}\left(\frac{ab\bar{x}+cd}{ab\bar{X}+cd}\right) - \bar{Y}\right]. \tag{13}$$

By using the equation (10), one can rewrite the above equation, as

$$Bias(t_3) = E\left[\bar{Y}(1+e_0)\left(\frac{ab\bar{X}(1+e_1)+cd}{ab\bar{X}+cd}\right) - \bar{Y}\right]$$

$$= E\left[\bar{Y}(1+e_0)(1+\eta e_1) - \bar{Y}\right]. \tag{14}$$

Rewrite equation (14) in term of equation (11) and (12), one get

$$Bias(t_3) = E\left[\bar{Y}(1+e_0+\eta e_1+\eta e_0e_1) - \bar{Y}\right]$$

$$= \bar{Y}E\left[e_0+\eta e_1+\eta e_0e_1\right]$$

$$= \frac{(1-f)}{n}\bar{Y}\eta CC_x^2. \tag{15}$$

Further, bias of t_4 and t_5 are obtained from equation (16) and (17) as follows:

$$Bias(t_4) = \frac{(1-f)}{n}g\eta\bar{Y}C_x^2\left[\frac{(g+1)}{2}\eta - C\right] \tag{16}$$

$$Bias(t_5) = \frac{(1-f)}{n}g\eta\bar{Y}C_x^2\left[\frac{(g-1)}{2}\eta + C\right]. \tag{17}$$

In addition, the MSE of t_3 can be found as follows:

$$MSE(t_3) = E(t_3 - \bar{Y})^2$$

$$= E\left[\bar{Y}(1+e_0)(1+\eta e_1) - \bar{Y}\right]^2$$

$$= \bar{Y}^2 E\left[e_0 + \eta e_1 + \eta e_0 e_1\right]^2$$

$$= \bar{Y}^2 E\left[e_0^2 + \eta^2 e_1^2 + 2\eta e_0 e_1\right]. \quad (18)$$

Rewrite equation (18) in term of equation (11) and (12), one get

$$MSE(t_3) = \bar{Y}^2 \left[\frac{(1-f)}{n} C_y^2 + \eta^2 \frac{(1-f)}{n} C_x^2 + 2\eta \frac{(1-f)}{n} C C_x^2\right]$$

$$= \frac{(1-f)}{n} \bar{Y}^2 \left[C_y^2 + \eta C_x^2 (\eta + 2C)\right]. \quad (19)$$

The MSE of t_4 and t_5 can be found from equations (20) and (21) as follows:

$$MSE(t_4) = \frac{(1-f)}{n} \bar{Y}^2 \left[C_y^2 + g\eta C_x^2 (g\eta - 2C)\right] \quad (20)$$

$$MSE(t_5) = \frac{(1-f)}{n} \bar{Y}^2 \left[C_y^2 + g\eta C_x^2 (g\eta + 2C)\right]. \quad (21)$$

The MSE of t_3, t_4, and t_5 in (19), (20), and (21) are, respectively, minimized for

$$\eta = -C = \eta_{(opt1.)} \quad (22)$$

$$\eta = C / g = \eta_{(opt2.)} \quad (23)$$

and

$$\eta = -C / g. \quad (24)$$

Therefore, the common minimum MSE of t_3, t_4, and t_5 is given by:

$$\min.MSE(t_3) = \min.MSE(t_4) = \min.MSE(t_5) = \frac{(1-f)}{n} \bar{Y}^2 \left[C_y^2 - C^2 C_x^2\right]. \quad (25)$$

A few estimators belonging to the estimators t_3, t_4, and t_5 for the convenience of the readers are given in the Table 1.

Table 1 A few members of t_3, t_4, and t_5

Estimator	Values of constants				
	a	b	c	d	g
A few members of t_3					
$t_{3(1)} = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$ The usual product estimator	1	1	1	1	-
$t_{3(2)} = \bar{y}\left(\frac{nC_x\bar{x}+\rho}{nC_x\bar{X}+\rho}\right)$ Yadav et al. (2019) estimator	n	C_x	ρ	1	-
$t_{3(3)} = \bar{y}\left(\frac{\beta_1(x)M_d\bar{x}+\rho C_x}{\beta_1(x)M_d\bar{X}+\rho C_x}\right)$ Yadav et al. (2019) estimator	$\beta_1(x)$	M_d	ρ	C_x	-
A few members of t_4					
$t_{4(1)} = \bar{y}$ (unbiased estimator)	a	b	c	d	0
$t_{4(2)} = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)$ Sisodia and Dwivedi (1981) estimator	1	1	C_x	1	1
$t_{4(3)} = \bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)$ Singh and Tailor (2003) estimator	1	1	ρ	1	1
A few members of t_5					
$t_{5(1)} = \bar{y}$ (unbiased estimator)	a	b	c	d	0
$t_{5(2)} = \bar{y}\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)$ Pandey and Dubey (1988) estimator	1	1	C_x	1	1
$t_{5(3)} = \bar{y}\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right)$ Singh and Tailor (2003) estimator	1	1	ρ	1	1

Efficiency Comparisons

In this section, the authors intend to compare the efficiency of t_3, t_4, and t_5 with other existing estimators as shown in Table 1.

It is well known that the MSE of unbiased estimator $\bar{y}$ under SRSWOR is given by

$$MSE(\bar{y}) = MSE(t_{4(1)}) = MSE(t_{5(1)}) = \frac{(1-f)}{n}\bar{Y}^2 C_y^2. \qquad (26)$$

The expressions for the MSE of a few members of t_3, t_4, and t_5 are derived as

$$MSE(t_{3(1)}) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + C_x^2(1+2C)\right] \qquad (27)$$

$$MSE(t_{3(2)}) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \eta_1 C_x^2(\eta_1 + 2C)\right] \quad (28)$$

$$MSE(t_{3(3)}) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \eta_2 C_x^2(\eta_2 + 2C)\right] \quad (29)$$

$$MSE(t_{4(2)}) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \eta_3 C_x^2(\eta_3 - 2C)\right] \quad (30)$$

$$MSE(t_{4(3)}) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \eta_4 C_x^2(\eta_4 - 2C)\right] \quad (31)$$

$$MSE(t_{5(2)}) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \eta_3 C_x^2(\eta_3 + 2C)\right] \quad (32)$$

$$MSE(t_{5(3)}) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \eta_4 C_x^2(\eta_4 + 2C)\right] \quad (33)$$

where $\eta_1 = \frac{nC_x\bar{X}}{nC_x\bar{X} + \rho}$, $\eta_2 = \frac{\beta_1(x)M_d\bar{X}}{\beta_1(x)M_d\bar{X} + \rho C_x}$, $\eta_3 = \frac{\bar{X}}{\bar{X} + C_x}$, $\eta_4 = \frac{\bar{X}}{\bar{X} + \rho}$.

It is observed from equation (25) to (33) that the estimators t_3, t_4, and t_5 are more efficient than

(i) the unbiased estimator $\bar{y}$ if

$$MSE(\bar{y}, t_{4(1)}, t_{5(1)}) - \min.MSE(t_3, t_4, t_5) = C^2C_x^2 > 0 \quad (34)$$

(ii) the estimator $t_{3(1)}$ if

$$MSE(t_{3(1)}) - \min.MSE(t_3, t_4, t_5) = (C+1)^2 > 0 \quad (35)$$

(iii) the estimator $t_{3(2)}$ if

$$MSE(t_{3(2)}) - \min.MSE(t_3, t_4, t_5) = \eta_1(\eta_1 + 2C) + C^2 > 0 \quad (36)$$

(iv) the estimator $t_{3(3)}$ if

$$MSE(t_{3(3)}) - \min.MSE(t_3, t_4, t_5) = \eta_2(\eta_2 + 2C) + C^2 > 0 \quad (37)$$

(v) the estimator $t_{4(2)}$ if

$$MSE(t_{4(2)}) - \min.MSE(t_3, t_4, t_5) = \eta_3 C_x^2(\eta_3 - 2C) + C^2C_x^2 > 0 \quad (38)$$

(vi) the estimator $t_{4(3)}$ if

$$MSE(t_{4(3)}) - \min.MSE(t_3, t_4, t_5) = \eta_4 C_x^2(\eta_4 - 2C) + C^2C_x^2 > 0 \quad (39)$$

(vii) the estimator $t_{5(2)}$ if

$$MSE(t_{5(2)}) - \min.MSE(t_3, t_4, t_5) = \eta_3 C_x^2(\eta_3 + 2C) + C^2 C_x^2 > 0 \qquad (40)$$

(viii) the estimator $t_{5(3)}$ if

$$MSE(t_{5(3)}) - \min.MSE(t_3, t_4, t_5) = \eta_4 C_x^2(\eta_4 + 2C) + C^2 C_x^2 > 0. \qquad (41)$$

Empirical Study

For empirical study, the authors have considered the data given in Yadav et al. (2019). The data belongs to a dataset showing the peppermint oil production from Siddhaur Block of Barabanki District at Uttar Pradesh State in India. The parameters of the population under consideration are given in Table 2. The dependent variable and the auxiliary variables are as follows:

Y : The production (Yield) of peppermint oil in kilogram

X : The area of the field in Bigha (2529.3 Square Meter)

Table 2 Parameters and constants of the population under study

$N = 150$	$n = 40$	$\bar{X} = 4.20,$	$\bar{Y} = 33.46$
$C_x = 0.73$	$C_y = 0.76$	$M_d = 3$	$\rho = 0.91$
$\beta_1(x) = 2.80$			

The authors have computed the percent relative efficiencies (PREs) of all existing estimators with respect to the unbiased estimator $\bar{y}$ for data statistics given in Table 2 and the findings are shown in Table 3.

Table 3 MSE and PREs of all existing estimators

Estimator	MSE	PRE
$\bar{y} = t_{4(1)} = t_{5(1)}$	12.9333	100.0000
A few members of t_3		
$t_{3(1)}$	47.4751	27.2423
$t_{3(2)}$	47.1335	27.4397
$t_{3(3)}$	46.6203	27.7418
t_3, t_4, t_5 (Proposed estimators)	2.2232	581.7336
A few members of t_4		
$t_{4(2)}$	2.3320	554.6029
$t_{4(3)}$	2.4111	536.4042
t_3, t_4, t_5 (Proposed estimators)	2.2232	581.7336
A few members of t_5		
$t_{5(2)}$	40.8552	31.6564
$t_{5(3)}$	39.5773	32.6786
t_3, t_4, t_5 (Proposed estimators)	2.2232	581.7336

From Table 3, one can derive two preliminary results as follows:

(i) For members of t_3 and t_5, it is envisaged that all estimators in these groups have quite similar MSE values except for the proposed estimator which has the smallest MSE values than other. Therefore, one can infer from the values of MSE that among all estimators, the proposed estimators at their optimum are the best in the sense of having the smallest MSE. However, when considering the values of PREs of all estimators, it has been found that the proposed estimators are always more efficient than the estimator $\bar{y}$ and other estimators.

(ii) For a member of t_4, it was observed that the estimator $t_{4(3)}$ had the largest MSE in comparison within its members of estimators in their group, as opposed to the proposed estimators. In terms of PREs, it was also found that the proposed estimators are better performing than any other estimators.

From these preliminary results, it can be inferred that the proposed estimators are more desirable overall the considered estimators under optimum conditions for this population data.

Conclusion

In this paper, three modified estimators based on known auxiliary information for estimating the population mean have been proposed by adapting the estimators of Yadav et al. (2019). The performance of the proposed estimators was compared with that of existing estimators using both a theoretical and an empirical study. It has been shown that under optimum conditions proposed estimators are better than other existing estimators that mentioned in the literature. Because they give a smaller MSE and a larger PRE when compared to the existing ones. Therefore, one can infer from the preliminary results that the proposed estimators is more desirable over all the existing estimators and should be put into practice.

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