DEA Model for Considering Relationship between Supply Chain Members

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Received: 21 January 2021; Revised: 23 December 2021; Accepted: 7 February 2022; Available online: 9 September 2022

Abstract

Background: A supply chain is a network system that consists of many key players from initial production to fulfilling customers' requirements. The performance of a supply chain can be measured by some variations of Network Data Envelopment Analysis (network DEA) which is a technique to measure the relative efficiency of a system, considering its internal structure. However, most variations of the network DEAs are not designed to include consideration of contract terms. Manufacturers often have contracts with suppliers for the long-term supply of their product. Such contracts are not easily terminated, modified or replaced. Alternative types of contracts, that do not bind the manufacturer to long-term commitments, can be quickly replaced by the manufacturer and/or supplier, to improve their supply chain performance. Therefore, the type of contracts that are extant or being considered is an important consideration when analyzing supply-chain performance. In this paper, a new network DEA, which can evaluate the efficiency of supply chains by considering the contract status, is proposed. **Methods:** By including the contract type in the DEA model in the representation of the internal structure of the supply chain, the efficiency of replaceable supplier contracts is acknowledged in the proposed methodology. This factor then contributes to the calculated efficiency score for the supply chain by comparing it with virtual chains that have the most efficient production capabilities. A virtual chain is generated by replacing an inefficient member with a more efficient one. To show the effectiveness and applicability of the proposed model, a case study of Thailand's processed food industry is used. **Results:** The results show that using the proposed model can identify inefficiencies in the supply chains by considering actual contract situations. It also can provide alternative instances of inefficient supply chains to help to achieve an efficient situation. **Conclusions:** The proposed model allows us to consider the efficiency of supply chains that include changeable suppliers who are themselves efficient. The case study used in the study was a processed food supply chain in Thailand. The results of the case study show that the proposed model can help assessors to understand their supply chain efficiency and also the effects of their suppliers' efficiency.

Keywords: Data Envelopment Analysis (DEA), Supply Chain, Food Industry

Introduction

Supply chain management (SCM) is a technique for integrating and managing activities which happen between every member in a chain. Intentions of introducing SCM are to satisfy customers requirements and achieve sustainable competitive advantages. One of the most important factors of SCM is cooperation among all members in a chain. Thus, for effective supply chain management, total efficiency of system should be measured as well as each member performance (Gunasekaran, Patel, & Tirtiroglu, 2001). To assess supply chains, adequate performance evaluation systems are required because it helps to evaluate current efficiency of supply chain and find out its weaknesses (Beamon, 1999).

Data envelopment analysis (DEA) is one of famous techniques for estimating efficiency of multiple decision-making units (DMUs) in various areas. The main approach of DEA is identifying the best practices of peer existed DMUs by comparing the DMUs' inputs and outputs ratio (Liang, Yang, Cook, & Zhu, 2006). Under the basic idea of the conventional DEA, each DMU is considered as a black-box, and the internal structure is not considered. To analyze complex structures, i.e. supply chain structures, multilevel DEA model and network DEA model have been developed (Fare & Grosskopf, 2000).

Generally, a supply chain consists of many types of members, i.e., suppliers and manufacturers. Each member is connected with others by relationships such as inter product flow and contact. From viewpoints of manufacturers, suppliers can be divided into two types, i.e., contacted one and non-contracted one (Chaowarat & Shi, 2013). Contracted suppliers have to provide products to clients, i.e., manufacturers, stably under the contract. Whereas non-contacted ones do not have such duty instead of the risks of losing orders. It means non-contracted suppliers need to improve their efficiency or they could be replaced by other suppliers by the manufacturers.

Supply chain consists of many members, an efficiency of supply chain depends on each member efficiency. To deal with supply chains which consist of many members, an alternative efficiency measurement methodology with network DEA model is proposed. The efficiency of replaceable supplier is contained in the proposed methodology to represent the internal structure. The proposed methodology helps to find out an efficiency score by comparing with virtual chains which have the best efficient production abilities. A virtual chain is generated by replacing an inefficient member with more efficient one. In this way, we can know the efficiency score after improving the weak point.

Background

Supply chain management and contractual scheme

Supply chain is a kind of network system in which many members work together to create final product from raw materials and then deliver the products to customers or serve required services to customers. One of important issues in supply chain management is cooperation of members. To achieve a successful supply chain management, all members should behave to optimize an entire chain's performance. However, members in a chain are self-interested entities and they tend to optimize their own production in actual situations. As a result, importance of whole chain performances tends to be downplayed. This causes a decreasing of entire chain's performance and profit. For this reason, a contractual scheme is introduced as coordination tools among members.

Sometime, contract systems are introduced into some supply chains to avoid risk from unstable production cost and investment. It is also brought to contain stability of production. Basically, there are two types of contracts between suppliers and manufacturers, contract and non-contact. The contract suppliers have agreements with the manufacturers that cover such issues as: conditions, price, volume, ordering periods, and warranty periods for selling products to manufactures. Usually, the manufacturers cannot replace the suppliers with others under the contracts during the term. On the other hand, the non-contact suppliers are easy to be replaced by manufacturers.

An actual supply chain is very complex. It may have an arbitrary number of members with very complicated network structure, each member has its own information about its production, and its activities may be unobserved by others. As a demonstrative example, an agricultural supply chain consisting of a manufacturers and farmers (suppliers). In this chain, the manufacturer is connected with farmers by inter product flow, i.e. crops and farmers contacts. It means contracted farmer has to deliver the crops to the manufacturers stable under the contract. On the other hand, non-contracted farmers need to improve their efficiency, or they could be changed by the manufacturers (Chaowarat, Suto, & Yokoi, 2015, Chaowarat, Suto, & Shi, 2014).

DEA models for supply chain

DEA is a linear programming, non-parametric technique used to measure relative efficiency of peer decision making units (DMUs) with multiple inputs and outputs (Amirteimoori & Khoshandam, 2011). In this method, a DMU is determined as efficient when it produces the maximum outputs and uses the minimum inputs.

The basic idea of DEA is using input and output values of other DMUs to construct a hypothetical composite DMU. If constructing a hypothetical composite DMU is possible, the in- considering DMU is decided as inefficient, otherwise it lies on the efficiency frontier and it is decided as efficient (Marti, Novakovi, & Baggia, 2009). The conventional DEA model is designed by considering only input and output values and internal structures of DMUs are ignored. There are two types of model, input oriented model and output oriented model. Under the input oriented model, all of the inputs are contracted as far as possible while controlling the outputs. On the other hand, under the output oriented model, the outputs are expanded as much as possible while consume the same amount of the inputs. The conventional DEA model for input-oriented can be denoted as follows:

$$\begin{aligned} \text{minimize} \quad & \theta \\ \text{subject to} \quad & \theta X_{io} - \sum_{j=1}^{n} \lambda_j X_{ij} \geq 0 \\ & \sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{ro} \\ & \lambda_j \geq 0 \end{aligned}$$

Where

θ : Efficiency score of the target DMU.

X_{io} : Vector of the i-th inputs of the target DMU.

X_{ij} : Vector of the i-th inputs of j-th DMU.

Y_{ro} : Vector of the r-th outputs of the target DMU.

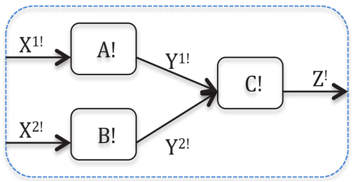
Y_{rj} : Vector of the r-th outputs of j-th DMU.

λ_j : Dual variable utilized to construct to a composite ideal DMU to dominate the target DMU.

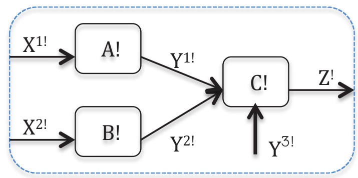
n : The number of DMUs.

For the supply chain management, DEA showed great promise to be a good evaluative tool, where the production function between the inputs and outputs was virtually absent or extremely difficult to acquire (Soheilirad et al.,2018). Especially, for the supply chain which contains many members and each member has its own input and output. George et. al. mentioned that "the standard DEA approach is a practical tool for efficiency evaluation however when there are more complex systems than a simple input-output procedure fails, to address the internal structures (Halkos, Tzeremes, & Kourtzidis, 2011)".

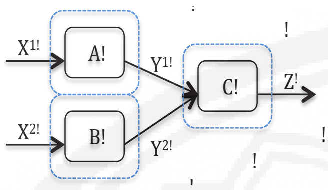
Usually, DMUs consist of many members and they have several structures, e.g. serial, parallel, and more complex ones (Monfared & Safi, 2013). However, we cannot evaluate DMUs which have a complex structure by using the conventional DEA. Hence, multilevel models have been proposed to solve this problem. Fare and Grosskopf's model (Fare & Grosskopf, 2000) is an example of the multilevel models. In this model, a structure of components in a target DMU is taken into consideration.



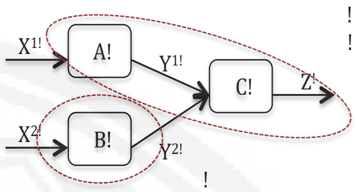
(a) Seiford's Model (Chen & Zhu, 2004)



(b) Chen's Model (Chen & Yan, 2011)



(c) Yang's Model (Yang, Wu, Liang, Bi, & Wu, 2011)



(d) Realistic situation

Figure 1 DEA models for supply chain

There are many researches that applies multilevel models for DEA to evaluate supply chain efficiency, for example,evaluating the performance of public pharmaceutical products supply chain by using three stage DEA (Chorfi & Berrado, 2019), measuring sustainable development in supply chain by using two stage DEA (Tsaples & Papathanasiou, 2020), evaluating efficiency in food industry supply chain by network-dynamic DEA (Kahi, Yousefi, Shabanpour, & Saen, 2017). However, there are still not that much researches that focus on the condition of members in the supply chain. So the DEA models that considering condition of members in the chain are reviewed.

Seiford et al. (1999) and Chen et al. (2004) used multilevel model for evaluating supply chains. Figure 1 illustrates the differences of structures of multilevel models for supply chains. In this figure, the dashed line rectangles and ellipses indicate production possibility sets. Production possibility set represents a unit of members which are evaluated the efficiency at one time. If there is only one production possibility set in a structure, it means all members in the structure are connected as an evaluand. On the other hand, if each member has its own production possibility set, it means that each member can be considered as an independent evaluand.

The most preferred supply chains consist of efficient members and they cooperate together to achieve the supply chain's goals. However, it is difficult to find such ideal supply chains, i.e., chains that consist of all efficient members, in practice. Thus, the ways to find inefficient members were required. Under the Seiford's model, each member in a supply chain is linked by intermediate products flow and all members cannot be replaced. The basic concept of Chen's model is same as the Seiford's model but an additional input is added in the second stage.

In these models, all members share a production possibility set to represent a situation that all members cannot be replaced. Structures of the Seiford's model and the Chen's model are shown in Figure 1 (a) and (b). We can evaluate static supply chains with this model. However, we cannot estimate the efficiency of the chains with considering situations in which some members are replaced to improve the chains performance.

In 2009, Yang et al. (2011) introduced a supply chain DEA model in which every member in a chain are considered as replaceable. Figure 1 (c). shows structure of supply chains assumed in the Yang's model. In this model, an overall supply chain efficiency is computed from efficiencies of all members. By using this model, all members efficiencies are also computed. Hence, we can create the ideal supply chain by choosing efficient members to compare with existing chains.

Figure 1 (d) illustrates a structure of chain that includes both types of suppliers, contract and non-contract, in a chain. To evaluate a supply chain in which some members are connected due to their contracts and some are not, the previous models cannot be used because members are not constantly linked together in the Zhu's model, and all members cannot be separated as in the Seiford's model. Thus, an alternative efficiency evaluation methodology which can be used to evaluate the above-mentioned structure supply chain is required. In this paper, Seiford's network DEA is extended and modified to respond the requirement.

Extended Supply Chain DEA model

For the simplicity, two stages supplier-manufacture chains which have two suppliers, and one manufacturer are considered. Fig. 2 illustrates the structure of the chains. The left side stands for supplier stage and the right side is manufacturer stage, where S1, S2 and M a contract supplier, a non-contract supplier and a manufacturer, respectively. X^1 and X^2 stand for inputs of supplier 1 (S1) and supplier 2 (S2) respectively. Intermediate products of S1 and S2 are indicated by Y^1 and Y^2 respectively. Z stands for a final output.

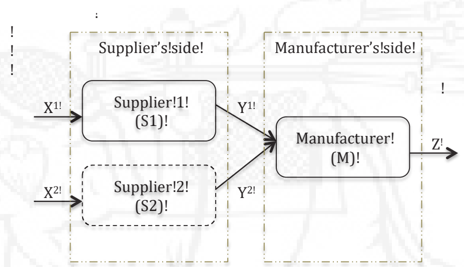


Figure 2 Two stages supplier-manufacturer supply chain

In this study, supply chains are discussed under two assumptions explained below:

Assumption 1;

CRS (Constant Returns to Scale) or CCR (named after its developer Chames, Cooper, & Rhodes, 1978) model is used to described supply chains, in which if the inputs increase, the outputs also increase in same proportion;

Assumption 2;

A non-contract supplier can be replaced with another supplier without extra cost or conditions.

Under CRS assumption, each supplier has a unique input-oriented projection in the production frontier by proportionally reducing inputs X_j while remaining same value outputs Y_j.

A non-contract supplier efficiency index

In some cases, non-contract suppliers are replaced with others to improve the supply chain efficiency if the suppliers do not have enough efficiency. In this research, the conventional CCR model is used to identify an efficiency of replaceable supplier (S2). Efficiency score of S2 can be identified by using following model.

$$
(S2)\left|\begin{array}{l}
\text{minimize } \theta_{s2} \\
\text{subject to } \theta_{s2} X_{io}^{2} - \sum_{j=1}^{n} \lambda_j^2 X_{ij}^2 \geq 0 \\
\qquad \sum_{j=1}^{n} \lambda_j^2 Y_{lj}^2 \geq Y_{ro}^2 \\
\qquad \lambda_j^2 \geq 0
\end{array}\right.
$$

A supply chain efficiency index

To obtain a supply chain's efficiency score, non-contract suppliers' efficiencies are computed, and the results are multiplied. The formulae are shown below:

$$
(R)\left|\begin{array}{l}
\text{minimize } \theta_R \\
\text{subject to } \theta_R X_{io}^1 - \sum_{j=1}^{n} \lambda_j^1 X_{ij}^1 \geq 0 \\
\qquad \sum_{j=1}^{n} \lambda_j^1 Y_{lj}^1 \geq Y_{lo}^1 \\
\qquad \theta_R X_{io}^2 - \sum_{j=1}^{n} \lambda_j^2 \theta_{s2} X_{ij}^2 \geq 0 \\
\qquad \sum_{j=1}^{n} \lambda_j^2 Y_{lj}^2 \geq Y_{lo}^2 \\
\qquad \sum_{j=1}^{n} \lambda_j^3 Y_{lj}^1 \leq Y_{lo}^1 \\
\qquad \sum_{j=1}^{n} \lambda_j^3 Y_{lj}^2 \leq Y_{lo}^2 \\
\qquad \sum_{j=1}^{n} \lambda_j^3 Z_{kj} \geq Z_{ko} \\
\qquad \lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0
\end{array}\right.
$$

where

θ_R : Efficiency score of the target chain.

θ_{s2} : Efficiency score of a changeable supplier.

X_{io}^1 : Vector of the i-th inputs of supplier 1 for the target chain.

X_{ij}^1 : Vector of the i-th inputs of supplier 1 for j-th chain.

Y_{lo}^1 : Vector of the l-th intermediate products of supplier 1 for the target chain.

Y_{lj}^1 : Vector of the l-th intermediate products of supplier 1for j-th chain.

X_{io}^2 : Vector of the i-th inputs of supplier 2 for the target chain.

X_{ij}^2 : Vector of the i-th inputs of supplier 2 for j-th chain.

Y_{lo}^2 : Vector of the l-th intermediate products of supplier 2 for the target chain.

Y_{lj}^2 : Vector of the l-th intermediate products of supplier 2 for j-th chain.

Z_{ko} : Vector of the k-th final outputs of manufacturer for the target chain.

Z_{kj} : Vector of the k-th final outputs of manufacturer for j-th chain.

$\lambda_j^1, \lambda_j^2, \lambda_j^3$: Dual variable utilized to construct to a composite ideal chain to dominate chain

n : The number of chains.

I : The number of supplier's inputs.

L : The number of intermediate products.

K : The number of final outputs.

Advantages of the proposed model

When a supply chain has mix of contract suppliers and non-contract suppliers, the existence models cannot show the efficiency based on the actual potential of the chain because they do not consider the fact that only non-contracted suppliers can be replaced. As a result, we may accredit inefficient supply chains as efficient. By using the proposed model, we can know efficiency scores under the conditions in which the possible improvements, i.e. replacing some inefficient and non-contracted suppliers with others, will be done.

In this section, advantages of the proposed model are shown. Efficiency scores obtained from the proposed model are discussed by comparing with three conventional models, i.e all members efficiency, the Seiford's network DEA, and the CCR model.

Comparison with the CCR model

In this section, advantages of the proposed model compared to the CCR model is discussed. When the efficiency score of a supply chain is less than 1, it indicates that the chain has possibilities to improve. Thus, the models which can find more chains which have an efficiency score of smaller than 1 can be said better to find the possibilities of supply chains.

Theorem 1:

The efficiency scores of supply chains obtained from the proposed model are not bigger than that obtained from CCR model which considers all processes as 'black box', that is,

$$\theta_R^* \leq \theta_T^*$$

Where

θ_T^* *: Efficiency score of a changeable supplier.*

$\theta_T^*, \lambda_j, j = 1, ..., n$ $\qquad$ $\lambda_j^{1'} = \lambda_j^{2'} = \lambda_j^{3'} = \lambda_j', \quad j = 1, ..., n.$

Proof: Suppose is an optimal solution to model R. Let

$\theta_T^*, \lambda_j^{1'}, \lambda_j^{2'}, \lambda_j^{3'}, j = 1, ..., n$

It is obvious that is also a feasible solution to model R. Thus, we have $\theta_R^* \leq \theta_T^*$, which completes the proof.

From the proof, it shows that values obtained from the proposed model are always smaller or equal to values obtained from the CCR model. It means the proposed model has more possible solution than the CCR model and we can detect an efficiency score more effective because it can identify the values that less than the values obtained from the CCR model.

Comparison with multiplication of each member's efficiency in supply chain

A chain consists of many members and each member has its own efficiency. The basic idea to compute efficiency score of chains is multiplying efficiency score of each member together. In this section, advantages of the proposed model in comparison with the basic idea is discussed. In this paper, efficiency score of each member is obtained by using the CCR model.

As mentioned in the previous section, the feasible region of the CCR model is smaller than the proposed model. On the same pace as the previous section, we prove that the proposed model feasible region is larger than the feasible region of the multiplication of each member's efficiency. To do this, the feasible regions of each member are considered together and compared with the proposed model's one. If the non-contract supplier is inefficient, its efficiency score is less than 1. To multiply the non-contract suppliers efficiency score to the proposed model feasible region, it will increase the feasible region. Therefore, the proposed model can find an efficiency score that is less than the result from the multiplication of each member's efficiency.

Theorem 2:

Efficiency scores obtained from the proposed model and each member efficiency have the following relationships.

$$\theta_R^* \leq \theta_{s1}^* * \theta_{s2}^* * \theta_m^*$$

Where

θ_{s1}: *Efficiency score of a constant supplier by the CCR model.*

θ_{s2}: *Efficiency score of a changeable supplier by the CCR model.*

θ_m: *Efficiency score of a manufacturer by the CCR model.*

Proof: Denote $\theta_{s1}^*, \lambda_j^{1*}; \theta_{s2}^*, \lambda_j^{2*}; \theta_m^*, \lambda_j^{3*}, j = 1, \ldots, n$ as the optimal pair of solutions corresponding to model (s1), (s2) and (m), respectively. And let P1 is:

$$\sum_{j=1}^{n} \lambda_j^{1*} X_{ij}^1 \leq \theta_{s1}^* X_{io}^1 \quad (1)$$

$$\sum_{j=1}^{n} \lambda_j^{1*} Y_{lj}^1 \geq Y_{lo}^1 \quad (2)$$

$$\sum_{j=1}^{n} \lambda_j^{2*} X_{ij}^2 \leq \theta_{s2}^* X_{io}^2 \quad (3)$$

$$\sum_{j=1}^{n} \lambda_j^{2*} Y_{lj}^2 \geq Y_{lo}^2 \quad (4)$$

$$\sum_{i=1}^{n} \lambda_j^{3*} Y_{lj}^1 \leq \theta_m^* Y_{lo}^1 \quad (5)$$

$$\sum_{j=1}^{n} \lambda_j^{3*} Y_{lj}^2 \leq \theta_m^* Y_{lo}^2 \quad (6)$$

$$\sum_{j=1}^{n} \lambda_j^{3*} Z_{kj} \geq Z_{ko} \quad (7)$$

(1) and (2) are taken from model $(s1)$. (3) and (4) are taken from model $(s2)$. (5), (6) and (7) are taken from model (m) . First, we prove that the feasible region of model (R) is greater than $(P1)$. By divide θ_{s1}^* on both side of (1), (2) and let $\lambda^{1*\prime} = \frac{\lambda^{1*}}{\theta_{s1}^*}$, divide θ_{s2}^* on both side of (3), (4) and let $\lambda^{2*\prime} = \frac{\lambda^{2*}}{\theta_{s2}^*}, \theta_m^*$ on both side of (5), (6), (7) and let $\lambda^{3*\prime} = \frac{\lambda^{3*}}{\theta_m^*}$ (To set (1), (5) and (6) in same structure in model (R) and compare only (2), (3), (4) and (7). Then $(P1)$ is:

$$\sum_{j=1}^{n} \lambda_j^{1*'} X_{ij}^1 \le X_{io}^1 \tag{8}$$

$$\sum_{j=1}^{n} \lambda_j^{1*'} Y_{lj}^1 \ge \frac{Y_{lo}^1}{\theta_{s1}^*} \tag{9}$$

$$\sum_{j=1}^{n} \lambda_j^{2*'} X_{ij}^2 \le X_{io}^2 \tag{10}$$

$$\sum_{j=1}^{n} \lambda_j^{2*'} Y_{lj}^2 \ge \frac{Y_{lo}^2}{\theta_{s2}^*} \tag{11}$$

$$\sum_{j=1}^{n} \lambda_j^{3*'} Y_{lj}^1 \le Y_{lo}^1 \tag{12}$$

$$\sum_{j=1}^{n} \lambda_j^{3*'} Y_{lj}^2 \le Y_{lo}^2 \tag{13}$$

$$\sum_{j=1}^{n} \lambda_j^{3*'} Z_{kj} \ge \frac{Z_{ko}}{\theta_m^*} \tag{14}$$

Now we multiply θ_{s2} to (10) to transform it as same structure in model (R), then we obtain that:

$$\sum_{j=1}^{n} \lambda_j^{1*'} X_{ij}^1 \le X_{io}^1 \tag{15}$$

$$\sum_{j=1}^{n} \lambda_j^{1*'} Y_{lj}^1 \ge \frac{Y_{lo}^1}{\theta_{s1}^*} \tag{16}$$

$$\sum_{j=1}^{n} \lambda_j^{2*'} \theta_{s2} X_{ij}^2 \le \theta_{s2} X_{io}^2 \tag{17}$$

$$\sum_{j=1}^{n} \lambda_j^{2*'} Y_{lj}^2 \ge \frac{Y_{lo}^2}{\theta_{s2}^*} \tag{18}$$

$$\sum_{j=1}^{n} \lambda_j^{3*'} Y_{lj}^1 \le Y_{lo}^1 \tag{19}$$

$$\sum_{j=1}^{n} \lambda_j^{3*'} Y_{lj}^2 \le Y_{lo}^2 \tag{20}$$

$$\sum_{j=1}^{n} \lambda_j^{3*'} Z_{kj} \ge \frac{Z_{ko}}{\theta_m^*} \tag{21}$$

To compare with the feasible region of (R), we obtain that:

$$\sum_{j=1}^{n} {}_j^{1*'} X_{ij}^1 \le X_{io}^1 \tag{22}$$

$$\sum_{j=1}^{n} \lambda_j^{1*'} Y_{lj}^1 \ge \frac{Y_{lo}^1}{\theta_{s1}^*} \ge Y_{lo}^1 \tag{23}$$

$$\sum_{j=1}^{n} \lambda_j^{2*'} \theta_{s2} X_{ij}^2 \le \theta_{s2} X_{io}^2 \le X_{io}^2 \tag{24}$$

$$\sum_{j=1}^{n} \lambda_j^{2*'} Y_{lj}^2 \ge \frac{Y_{lo}^2}{\theta_{s2}^*} \ge Y_{lo}^2 \tag{25}$$

$$\sum_{j=1}^{n} \lambda_j^{3*'} Y_{lj}^1 \le Y_{lo}^1 \tag{26}$$

$$\sum_{j=1}^{n} \lambda_j^{3*'} Y_{lj}^2 \le Y_{lo}^2 \tag{27}$$

$$\sum_{j=1}^{n} \lambda_j^{3*'} Z_{kj} \ge \frac{Z_{ko}}{\theta_m^*} \ge Z_{ko} \tag{28}$$

The connect the above facts, we see that the feasible region of (R) is greater than$(P1)$. Thus, $\theta_R^* \le \theta_{s1}^* * \theta_{s2}^* * \theta_m^*$.

As we can see from above formula, the proposed model always finds values that smaller or equal to the multiplication of each member efficiency. It means the proposed model can detect inefficiency supply chain while the multiplication of each member efficiency cannot.

Comparison with the Seiford's network DEA model

As mentioned in the previous section, the Seiford's network DEA is presented to identify supply chains' performance by considering a chain as a network system, and each member in a chain is linked by internal products. But in the actual situation, there are a supply chain that there are mix of contract and non-contract supplier that cannot be linked all the time. In this type of chain, the Seiford's network DEA is not suit for evaluating the chain performance. Its feasible region is limited due to its restriction that all members must be linked. To compare with the Seiford's network DEA model, the proposed model is designed to determine chain efficiency in the feasible region that only contract suppliers are fixed. Therefore, the proposed model has more opportunity to find better solution than the Seiford's network DEA models. Thus the efficiency scores obtained from the proposed model is always less or equal than the obtained from the Seiford's network DEA models.

Theorem 3:

Efficiency from the proposed model and efficiency from the Seiford's network DEA model [Fare & Grosskopf 2000] have the following relationship.

$$\theta_R^* \leq \theta_{Network}^*$$

Where

$\theta_{Network}^*$: *Efficiency score of the target chain by the network DEA model.*

Proof: Let $\theta_{Network}^*, \lambda_j^{1*}, \lambda_j^{2*}, \lambda_j^{3*}, j = 1, \ldots, n$ be any optimal of the network DEA model. That is:

$$\sum_{j=1}^{n} \lambda_j^{1*} X_{ij}^1 \leq \theta_{Network}^* X_{io}^1 \tag{29}$$

$$\sum_{j=1}^{n} \lambda_j^{1*} Y_{lj}^1 \geq Y_{lo}^1 \tag{30}$$

$$\sum_{j=1}^{n} \lambda_j^{2*} X_{ij}^2 \leq \theta_{Network}^* X_{io}^2 \tag{31}$$

$$\sum_{j=1}^{n} \lambda_j^{2*} Y_{lj}^2 \geq Y_{lo}^2 \tag{32}$$

$$\sum_{j=1}^{n} \lambda_j^{3*} Y_{lj}^1 \leq Y_{lo}^1 \tag{33}$$

$$\sum_{j=1}^{n} \lambda_j^{3*} Y_{lj}^2 \leq Y_{lo}^2 \tag{34}$$

$$\sum_{j=1}^{n} \lambda_j^{3*} Z_{kj} \geq Z_{ko} \tag{35}$$

To compare with the feasible region of (R), θ_{s2} is multiplied to (31) and (32) and let $\bar{\lambda}_j^{2*} = \hat{\theta}_{s2}^* * \lambda_j^{2*}$. That is:

$$\sum_{j=1}^{n} \lambda_j^{1*} X_{ij}^1 \leq \theta_{Network}^* X_{io}^1 \tag{36}$$

$$\sum_{j=1}^{n} \lambda_j^{1*} Y_{lj}^1 \geq Y_{lo}^1 \tag{37}$$

$$\sum_{j=1}^{n} \bar{\lambda}_j^{2*} X_{ij}^2 \leq \theta_{Network}^* \theta_{s2} X_{io}^2 \tag{38}$$

$$\sum_{j=1}^{n} \bar{\lambda}_j^{2*} Y_{lj}^2 \geq \theta_{s2} Y_{lo}^2 \quad (39)$$

$$\sum_{j=1}^{n} \lambda_j^{3*} Y_{lj}^1 \leq Y_{lo}^1 \quad (40)$$

$$\sum_{j=1}^{n} \lambda_j^{3*} Y_{lj}^2 \leq Y_{lo}^2 \quad (41)$$

$$\sum_{j=1}^{n} \lambda_j^{3*} Z_{kj} \geq Z_{ko} \quad (42)$$

(36), (37), (40), (41) and (42) are same as in $(Network)$. Because $0 < \theta_{s2} \leq 1$ from model $^{(s2)}$, comparing (31) and (32) with $(Network)$, that is:

$$\sum_{j=1}^{n} \bar{\lambda}_j^{2*} X_{ij}^2 \leq \theta_{Network}^* \theta_{s2} X_{io}^2 \leq \theta_{Network}^* X_{io}^2 \quad (43)$$

$$\sum_{j=1}^{n} \bar{\lambda}_j^{2*} Y_{lj}^2 \geq Y_{lo}^2 \geq \theta_{s2} Y_{lo}^2 \quad (44)$$

(36) - (42) and (43), (44) imply that the feasible region of (R) is greater than $(Network)$. Thus $\theta_R^* \leq \theta_{Network}^*$.

As we can see from above formula, the feasible region of the proposed model is bigger than the Seiford's network DEA model. As consequence, the proposed model has more space to search an optimal solution. Therefore, we can say that with the proposed model we can get more practical solutions than with the Seiford's network DEA models.

Case study

In this section, a processed food industry case is presented as example to demonstrate applicability of the proposed model. In this case, manufacturers receive raw materials from many suppliers to reduce the risks come from unstable productivity. Some of the suppliers have a long-term contract of supplying with manufactures and the others do not have such contracts.

Table 1 Data sets of the samples

DMUs		FA (10^6 THB)	PC (10kg./3.95 Acres)	ITC (%)	DR (%)	SOR (%)	ROA	RP (%)
1	S1	41.94	198.00	49.33	4.75	92.00	6.25	87.50
	S2	5.36	198.32	48.30	7.80	90.20		
2	S1	38.37	241.62	42.80	6.00	94.82	14.97	96.22
	S2	4.56	251.23	35.50	7.31	88.20		
3	S1	33.22	350.00	35.62	6.07	93.25	2.04	97.45
	S2	6.58	306.90	42.70	5.56	89.90		
4	S1	36.69	209.74	47.97	12.40	95.93	7.70	96.42
	S2	7.67	271.30	44.20	4.90	90.20		
5	S1	19.53	354.13	50.00	3.23	94.95	6.56	93.02
	S2	7.95	373.80	47.00	6.78	91.83		
6	S1	41.53	230.38	42.23	10.21	96.41	11.32	86.03
	S2	8.23	282.80	42.50	3.70	88.40		

Table 1 (Cont.)

DMUs		FA (10^6 THB)	PC (10kg./3.95 Acres)	ITC (%)	DR (%)	SOR (%)	ROA	RP (%)
7	S1	10.44	163.00	41.12	6.48	95.95	13.66	95.66
	S2	5.30	275.80	35.50	4.38	75.60		
8	S1	11.99	322.00	38.58	7.01	94.96	13.91	96.74
	S2	6.24	328.70	39.80	2.96	79.50		
9	S1	15.04	289.26	36.73	0.98	95.49	2.84	88.65
	S2	4.89	291.30	37.00	2.75	92.60		
10	S1	16.43	274.61	30.00	7.84	95.10	7.08	96.07
	S2	8.23	350.50	40.50	3.79	95.00		
11	S1	27.70	250.00	42.32	3.47	94.74	16.18	91.50
	S2	7.45	306.90	49.50	3.25	90.50		
12	S1	16.35	213.00	37.27	6.73	94.61	5.28	96.50
	S2	3.03	160.90	42.50	4.15	85.40		
13	S1	36.74	319.23	35.00	6.00	94.47	8.24	94.75
	S2	2.51	158.80	46.50	4.63	65.30		
14	S1	13.45	269.00	45.95	8.00	95.70	3.61	90.50
	S2	4.23	209.80	36.00	2.90	70.60		
15	S1	20.28	295.21	37.00	4.78	94.41	3.50	96.53
	S2	6.12	271.30	43.50	3.01	82.50		

In this case study, the suppliers are farmers in aquaculture and the manufacturers are factories that produce seafood products such as canned fruit, ready-to-eat food. The factories receive raw materials of vegetable and meat, e.g., pineapple, corn or shrimp, from farmers. Qualities of the raw materials of vegetable and meat products depend on the farmers. Factories try to choose the farmers that can provide them good quality materials. High performance farmer could get a contract of trading for a fix-period. Meanwhile non-contract farmers need to improve their efficiency otherwise they could be replaced by the factories. Under the consumption that non-contract farmers can be replaced without any constraint conditions. For example, if the manufacturer wants to change its non-contract farmers, the manufacturer do not need to pay for any additional cost.

Table 1 shows a data set of supply chain obtained from processed food industry in Thailand (Chaowarat, 2014). All of the companies listed in the table are members of Thailand Institute of Scientific and Technological Research (TISTR), Food Technology Department. An efficiency analysis of all companies is performed using data for year 2019.

In the supplier stages, both suppliers consume some inputs; Fixed Assets (FA), Production Capacity (PC) and Inventory Transportation Cost (ITC) to generate Damage Rate (DR) and Supplier On-time Rate (SOR). In the manufacturer stages, DR and SOR are used to produce Returns On Assets (ROA) and Returned Products (RP).

Table 2 reports the efficiency scores of the non-contracted suppliers and supply chain obtained from the conventional DEA model, the Seiford's network DEA model and the proposed model. In the first column, we can find that the non-contracted suppliers of chain 1, 2, 8, 10, 11, 13, 14 and 15 are efficient, it means only seven suppliers are efficient in the fifteen suppliers. The results show that there are five inefficient chains by using the conventional DEA, while there nine inefficient chains by using network DEA model. The results from the proposed method show ten chains are inefficient. From the results, we can conclude that:

(i) The supply chain efficiency scores from the proposed model are not larger than the efficiency scores from the conventional DEA and the Seiford's network DEA. This result shows that the proposed model can find inefficient chains more sensitively than the two conventional models.

(ii) The proposed model provides an alternative instance for an inefficient supply chain to improve its suppliers. For example, we can know that chain 15 can be improved because the changeable supplier is inefficient (based on suppliers of chain 4,9,14 and 15). Consequently, chain 15 also can be improved the same manner as chain 4, 9, 14 and 15.

Table 2 Efficiency results

DMUs	θ_{S2} By S2	θ By Conventional DEA	θ_N By NDEA	θ_R By the proposed model
1	1.000	1.000	1.000	1.000
2	1.000	0.901	0.921	0.906
3	0.856	1.000	1.000	0.952
4	0.897	1.000	0.906	0.906
5	0.782	1.000	0.948	0.946
6	0.889	1.000	1.000	1.000
7	1.000	0.854	0.910	0.900
8	0.913	0.939	0.910	0.898
9	1.000	1.000	1.000	1.000
10	1.000	1.000	1.000	1.000
11	0.822	1.000	0.961	0.961
12	1.000	0.915	0.915	0.912
13	1.000	1.000	0.931	0.920
14	1.000	1.000	1.000	1.000
15	0.882	0.970	0.917	0.917

Conclusion and Suggestions

This paper presents an extended network DEA model for supply chain efficiency measurement by considering replaceable suppliers. Two types of suppliers, contract supplier and the non-contract supplier, are included in the proposed model. Unlike the conventional models, the proposed model allows us to consider efficiency of supply chains in cases that their changeable suppliers are efficient. As a case study, a case of processed food supply chain in Thailand is shown. The results of the case study shows that the proposed model can help assessors to understand their supply chain efficiency and effects of their suppliers' efficiency.

In this paper, we assumed that non-contract members can be replaced without any constraint conditions. But in actual supply chains, there are additional conditions that will be occurred when replace a non-contract supplier. For example, when a manufacturer replaces suppliers, it has to pay extra costs, e.g., cutting contract cost, opportunity lost cost while seeking new suppliers. Therefore, adding such conditions into the model could be another direction for future work in this area. In some cases, outputs of supply chain are considered because usually to reduce or increase inputs is difficult. The proposed models are under the assumption of input-orient models which is focused on decrease inputs and keep same number of outputs. In some cases, some inputs already reach their minimum limit. In such cases, increasing some outputs should be considered. To adapt such cases, output-oriented model also should be developed in the future. In the proposed model, only the fixed structure, i.e., there are two suppliers and a manufacture, and one supplier has contract, and another does not have. To apply the model to more complex situations, the structure must be expanded in the future. In this paper, the case study of food industry is used to show a usage of the proposed model due to its supply chain structure. The proposed model is also possible to apply with other industries that they have the same structure.

References

Amirteimoori, A., & Khoshandam, L. (2011). A data envelopment analysis approach to supply chain efficiency. *Advances in Decision Sciences*, *2011*, 1-8.

Beamon, B. (1999). Measuring supply chain performance. *International Journal of Operations & Production Management*, *19*(3), 2752-2792.

Chaowarat, W. (2014). *Efficiency Evaluation of Supply Chain using Data Envelopment Analysis (DEA) Considering Contract Relationship between Supplier and Manufacturer Case studies on Thai frozen food industry*. Japan: Muroran Institute of Technology.

Chaowarat, W., & Shi, J. (2013). Efficiency measurement of supply chain in the export frozen vegetable industry: A network dea approach. *International Journal of Materials, Mechanics and Manufacturing*, *1*(1), 88-91.

Chaowarat, W., & Sopadang, A. (2011). *Performance measurement of supply chain management for export frozen food*. Chiang Mai University, Thailand.

Chaowarat, W., Suto, H., & Shi, J. (2014). An evaluation method of supply chain efficiency considering information sharing level. *IEEJ Transactions on Electronics, Information and Systems*, *134*(11), 1640-1646.

Chaowarat, W., Suto, H., & Yokoi, T. (2015). An evaluation method of supply chain efficiency considering customers' satisfaction. *Transaction of Japan Society of Kansei Engineering*, *14*(1), 231-237.

Charnes, A., Cooper, W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*, *2*(6), 429-444.

Chen, C., & Yan, H. (2011). Network DEA model for supply chain performance evaluation. *European Journal of Operational Research*, *213*(1), 147 - 155.

Chen, Y., & Zhu, J. (2004). Measuring information technology's indirect impact on firm performance. *Information Technology and Management*, *5*, 9-22.

Chorfi, Z., & Berrado, A. (2019). A three stage dea approach for evaluating the performance of supply chains: Case study of public pharmaceutical products supply chains in morocco. Proceedings of the International Conference on Industrial Engineering and Operations Management March 5-7, 2019 (pp. 278-288). Bangkok: Thailand.

Fare, R., & Grosskopf, S. (2000). Network DEA. *Socio-Economic Planning Sciences*, *34*(1), 35 - 49.

Gunasekaran, A., Patel, C., & Tirtiroglu, E. (2001). Performance measure and metrics in a supply chain environment. *International Journal of Operations & Production Management*, *21*, 71-87.

Halkos, G., Tzeremes, N., & Kourtzidis, S. (2011). *The use of supply chain dea models in operations management: A survey*. Retrieved from https://mpra.ub.uni-muenchen.de/31846/

Kahi, V. S., Yousefi, S., Shabanpour, H., & Saen, R. F. (2017). How to evaluate sustainability of supply chains? a dynamic network dea approach. *Industrial Management Data Systems*, *117*(9) 1866-1889.

Liang, L., Yang, F., Cook, W., & Zhu, J. (2006). Dea models for supply chain efficiency evaluation. *Annals of Operations Research*, *145*(1), 35-49.

Marti, M., Novakovi, M., & Baggia, A. (2009). Data envelopment analysis - basic models and their utilization. *Organizacija*, *42*(2), 37-43.

Monfared, M., & Safi, M. (2013). Network dea: an application to analysis of academic performance. *Journal of Industrial Engineering International*, *9*(1) 1-10.

Seiford, L., & Zhu, J. (1999). Profitability and marketability of the top 55 us commercial banks. *Management Science*, *45*(9), 1270-1288.

Soheilirad, S., Govindan, K., Mardani, A., Zavadskas, E. K., Nilashi, M., & Zakuan, N. (2018). Application of data envelopment analysis models in supply chain management: a systematic review and meta-analysis. *Annals of Operations Research*, *271*(2), 915-969.

Tsaples, G., & Papathanasiou, J. (2020). Multi-level dea for the construction of multi-dimensional indices. *MethodsX*, *7*, 101169.

Yang, F., Wu, D., Liang, L., Bi, G., & Wu, D. (2011). Supply chain DEA: production possibility set and performance evaluation model. *Annals of Operations Research*, *185*, 195-211.