



# Exchange-field-controllable 0- $\pi$ transition in asymmetric Josephson spin-valve heterostructure

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## Abstract

The Josephson Effect in an asymmetric SIFM<sub>1</sub>IFM<sub>2</sub>IS Josephson Junction (JJ) in which the two SIF bilayers have different ferromagnetic layers (FM<sub>1</sub> and FM<sub>2</sub>) is studied. The linearized Usadel equations, which are valid for low transparency interfaces between S and FM layers, are used to obtain the expressions for the critical current in this junction. The relative magnitudes and directions of the exchange fields in (FM<sub>1</sub> and FM<sub>2</sub>) are treated as adjustable parameters. The Josephson currents in this junction are simulated by numerically evaluating the expressions for various values of the adjustable parameters. We consider the cases of the exchange fields in the two ferromagnetic layers being parallel and being antiparallel. We show that increasing the exchange field in the first layer of a parallel aligned junction while maintaining the value of the exchange field in the second layer will induce a “0- $\pi$ ” transition, i.e., a reversal of the direction of the current, but that increasing the exchange field in an antiparallel alignment will not. When the exchange field in the second layer in a parallel aligned SIFM<sub>1</sub>IFM<sub>2</sub>IS junction is large, it is seen that a small change of the exchange field in the first layer can induce the switch of “0-state” JJ to a “ $\pi$ -state” JJ in the entire temperature range of operation ( $0 < T < T_c$ ) of the junction. PACS classification codes: 74.80.Dm; 74.50.+r; 75.30.Et; 74.60.Jg

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## Introduction

Bulaevskii, Kuzil, and Sobyanin (1977) predicted that under certain conditions, the direction of the critical current in a tunnel Josephson junction containing magnetic impurities in the insulating layer would be reversed. This would lead to a negative Josephson coupling. Buzdin, Bulaevskii, and Panjukov (1982) suggested that similar results could be duplicated if the insulating layer was replaced by a ferromagnetic (FM) layer. Buzdin (2005) explained this behavior as the result of the difference in the phase of the two superconducting layers being equal to  $\pi$ . When this occurs, the Josephson junction (JJ) is called a “ $\pi$  JJ”. When the phase difference is 0, the junction is called a “0 JJ”. Since current-voltage measurements can only provide information on the absolute value of the critical current,  $|J_c|$ , these measurements can not be used to distinguish between a single “0 JJ” and a single “ $\pi$  JJ”. Additional steps such as embedding the SFS “JJ” in a superconducting loop (Dayton et al., 2018; Pfeiffer et al., 2008; Li et al., 2019) or using a dc SQUID interferometry (Li et al., 2008) must be taken. Ryzaznov et al. (2001) were among the first to obtain a “ $\pi$  JJ”.

When the direction of the current can be reversed by changing some of the parameters of the junction, the junction is called a “0- $\pi$ ” junction. (Pfeiffer et al., 2008; Born et al., 2006; Frolov, Van Harlingen, Boginov, Oboznov, & Ryazanov, 2006; Gingrich et al., 2016) In their study, Born et al. (2006) have seen evidence of F-layer thickness inducing 0- $\pi$  transitions in the critical currents  $I_c(d_F)$  of the SFS junctions. They observed six damped oscillations in the critical current in the junctions as the thickness of the ferromagnetic layer was increased. They attributed the behavior to the presence of three 0- $\pi$  transitions. Frolov et al. (2006) obtained



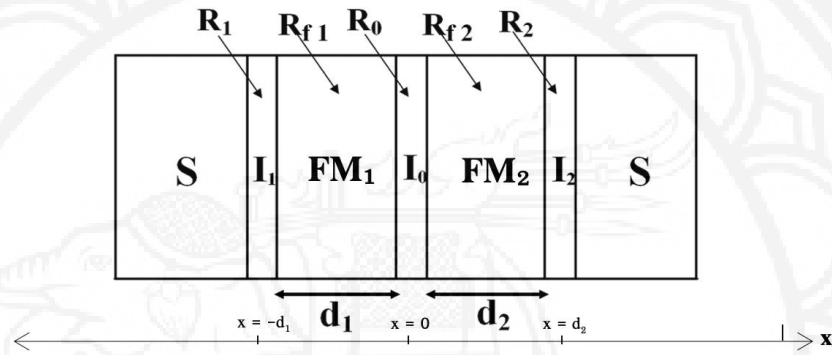
the “0- $\pi$ ” transition while they were varying the effective barrier thickness of the ferromagnetic layer in a SFS “JJ”. Pfeiffer et al. (2008) emphasized the importance of there being an insulating layer in the SFS “JJ” (leading to an SIFS “JJ”). The insulating layer produces a resistance  $R$  which can be tuned to have large values by varying the thickness of the insulating layer. The need to have high resistance–area products  $R \times A$  is the requirement that the responses have low damping. Radovic, Lazarides, and Flytzanis (2003) found that the transition from a “0-JJ” to “ $\pi$  JJ” could be achieved by varying the temperature.

Koshina and Krivoruchko (2001) considered another type of Josephson junctions, ones which had two ferromagnetic layers in them (the  $SF_1F_s$  “JJ”). The behaviors of these “JJ’s” will depend on the relative alignment of the two exchange fields in the two FM layers. Bergeret, Volkov, and Efetov (2001) Bergeret, Volkov, and Efetov (2005) Buzdin (2003) have investigated how the alignment of the magnetization in the two ferromagnetic layers can affect the Josephson current in these junctions. In the case of parallel alignment, a transition from the 0-state to the  $\pi$ -states can occur as the exchange fields are increased. However, when the alignment is antiparallel, the transition can not be induced by changing the value of the exchange fields. In addition to the change in the sign, the conductance of a junction with antiparallel alignment will increase when the exchange field in  $SF_1F_s$  junction with thin FM-layers increases. In the junctions with parallel alignment, the conductance will decrease as the exchange field increases. Golubov, Kupriyanov, and Fominov (2002) have studied the density of state for spin up electrons in the FM-layers of the  $SF_1F_s$  junctions. They also calculated the critical currents of these  $SFM_1FM_2S$  junction. Bergeret et al. (2005) point out that for very thin FM and S layers, the SF bilayer can be treated as a single ferromagnetic superconductor (Fs). This would lead to the  $SF_1F_s$  junction becoming a  $F_s/I/F_s$  junction (Li, Zheng, Xing, Sun, & Dong, 2002). Study of this type of junction will be the subject of another paper such as superconducting spin valve (Linder & Robinson, 2015).

In the present study, we consider double ferromagnetic barrier junctions similar to ones considered (Bergeret et al., 2001; Karabassov et al., 2020; Satchell et al. (2020)) except for the fact that FM and S layers are not thin and junctions have thin insulating barriers placed at all interfaces. The conclusions reached in Radovic et al., (2003) and other studies on different types of junctions were based on the tunneling Hamiltonian approach in which strong barrier strength was assumed. This may not be true for very thin insulating layers. In this work, the Josephson Effect in a  $SIFM_1IFM_2IS$  junction is studied by solving analytically the Usadel equation for the case of low interface transparency (Buzdin, 2005). We find that a transition from a parallel aligned 0-state junction into a  $\pi$ -state junction can be induced by changing the relative orientation of magnetic moment in the  $FM_1$ -layer, but that a transition from an antiparallel aligned  $\pi$ -state junction into the 0-state can not be induced by changing the orientation. For the case of junctions having parallel alignment of the magnetizations in the FM-layers, we found that the critical current can be enhanced by increasing the exchange field. However, when the exchange fields are antiparallel, the critical currents can not be increased by increasing the exchange field. We have calculated the ratio  $R$  of the difference between the critical currents of those junctions having parallel and antiparallel alignments of the magnetizations in the two FM-layers to the critical currents of junction having antiparallel alignment of the magnetizations.

### Theoretical framework

Our model of the SIFM<sub>1</sub>IFM<sub>2</sub>IS junction is based on the following assumptions. First, the S/FM interface is assumed to have low transparency. Therefore the anomalous Green function in each ferromagnetic layer will be small allowing the linearized Usadel equation to be used. Second, the energy gap of each superconductor is constant and equal to  $\Delta_{\pm} = \Delta e^{\pm i\frac{\phi}{2}}$ , where the  $\pm$  signs refer to the right and the left superconductors respectively. Third, the energy gap vanishes in the ferromagnetic regions and there is no penetration of the ferromagnetic exchange energy into the superconducting region. Finally, because of the low S/FM interface transparency, there is no proximity effect in the sandwich ( $d \gg \xi$ ). Also, there is no leakage of the superconducting order parameter into the ferromagnetic layer. The FM<sub>1</sub> and FM<sub>2</sub> are taken to be thin ferromagnetic layers of thicknesses  $d_1$  and  $d_2$  respectively. (See Figure 1).



**Figure 1** Schematic of the SIFM<sub>1</sub>IFM<sub>2</sub>IS Junction. The positions of the thin insulating layers inserted into a SFM<sub>1</sub>FM<sub>2</sub>S junction and of the resistances are indicated

We assume that bulk superconductors are in the dirty limit. Here,  $x$  is the coordinate along the normal direction to the interface,  $R_1$ ,  $R_2$  and  $R_0$  are the interface barriers resistance per unit area located at  $x = -d_1$ ,  $d_2$  and 0 respectively,  $R_{f1}$  is FM<sub>1</sub>-layer resistance per unit area and  $R_{f2}$  is FM<sub>2</sub>-layer resistance per unit area. In the case of the low interface transparency, we use the following linearized Usadel equation (Buzdin, 2005),

$$-\frac{D_f}{2} \frac{\partial^2}{\partial x^2} F_i + [\omega + i \text{sgn}(\omega) h_i] F_i = 0, \quad (1)$$

where  $\omega = (2n + 1)\pi T$  are the Matsubara Frequencies,  $h_i$  is the ferromagnetic exchange field acting on the spin of an electron in the ferromagnetic layer 'i' and  $D_f$  is the electron diffusion coefficient in ferromagnetic layers.  $F_1$  and  $F_2$  are the anomalous Green function in FM<sub>1</sub> and FM<sub>2</sub> layers, respectively. The solution of Equation. (1) can be written in the following form,

$$\text{for } -d_1 < x < 0, \quad F_1(x) = A_1 e^{k_1 x} + A_2 e^{-k_1 x} \quad (2)$$

and

$$\text{for } 0 < x < d_2, \quad F_2(x) = B_1 e^{k_2 x} + B_2 e^{-k_2 x}, \quad (3)$$

$$\text{where } k_{1(2)} = \sqrt{\frac{2}{D_{f_{1(2)}}} [\omega + i \text{sgn}(\omega) h_{1(2)}]} \quad \text{and} \quad \xi_{n1(2)} = \sqrt{\frac{D_{f_{1(2)}}}{\pi T_c}} \quad \text{with the indices 1, 2 refer to FM}_1 -$$



and FM<sub>2</sub>-layers, respectively. The boundary conditions at the interfaces of this junction for the Usadel equation are given as (Buzdin, 2005; Bergeret et al., 2005)

$$\gamma_1 \frac{\partial}{\partial x} F_1 \Big|_{x=-d_1} = -G_s \left\{ \frac{\Delta_-}{\omega} - F_1(-d_1, \omega) \right\} \text{sgn}(\omega), \quad (4)$$

$$\gamma_2 \frac{\partial}{\partial x} F_2 \Big|_{x=d_2} = +G_s \left\{ \frac{\Delta_+}{\omega} - F_2(d_2, \omega) \right\} \text{sgn}(\omega), \quad (5)$$

$$\alpha_1 \frac{\partial}{\partial x} F_1 \Big|_{x=0} = F_2(0) - F_1(0) \quad (6)$$

and

$$\alpha_2 \frac{\partial}{\partial x} F_2 \Big|_{x=0} = F_2(0) - F_{f1}(0), \quad (7)$$

where

$$\gamma_1 = \frac{R_1}{R_{f1}} d_1, \quad \gamma_2 = \frac{R_2}{R_{f2}} d_2$$

and

$$\alpha_1 = \frac{R_0}{R_{f1}} d_1, \quad \alpha_2 = \frac{R_0}{R_{f2}} d_2. \quad (8)$$

Here, the normal Green function in the superconducting layer is  $G_s = \frac{\omega}{\sqrt{\omega^2 + \Delta^2}}$ , where  $\Delta_{\pm} = \Delta e^{\pm i\frac{\varphi}{2}}$ .

Solving the above boundary value problem, we obtain

$$F_1(x, \omega, h) = \frac{\Delta}{\omega} \cdot \frac{|G_s|}{\chi} \left[ \begin{aligned} &\left\{ A_{1-} \cos(\varphi/2) - A_{1+} i \sin(\varphi/2) \right\} e^{k_1 x} \\ &- \left\{ A_{2-} \cos(\varphi/2) - A_{2+} i \sin(\varphi/2) \right\} e^{-k_1 x} \end{aligned} \right], \quad (9)$$

where

$$A_{1\pm} = B_{1-} \pm B_{2+}, \quad A_{2\pm} = B_{1+} \pm B_{2+},$$

$$B_{1\pm} = M_{2+} - M_{2-} + [M_{2-} N_{2-} - M_{2+} N_{2+}] N_{1\pm}, \quad B_{2\pm} = M_{1\pm} [N_{2-} - N_{2+}],$$

$$M_{1\pm} = (|G_s| \pm \gamma_1 k_1) e^{\pm k_1 d_1}, \quad M_{2\pm} = (|G_s| \pm \gamma_2 k_2) e^{\pm k_2 d_2}, \quad N_{1\pm} = 1 \pm \alpha_1 k_1, \quad N_{2\pm} = 1 \pm \alpha_2 k_2$$

and

$$\begin{aligned} \chi = & M_{1+} M_{2+} [N_{1+} N_{2+} - 1] + M_{1-} M_{2-} [N_{1-} N_{2-} - 1] \\ & - M_{1+} M_{2-} [N_{1+} N_{2-} - 1] - M_{1-} M_{2+} [N_{1-} N_{2+} - 1] \end{aligned} \quad (10)$$

To derive the expressions for the super current for FM<sub>1</sub>-layer, we use the following definition (Buzdin, 2003)

$$I_s(\varphi) = \frac{i\pi T \sigma_f}{2e} \sum_{\omega=-\infty}^{\infty} \left( \tilde{F} \frac{\partial}{\partial x} F - F \frac{\partial}{\partial x} \tilde{F} \right), \quad (11)$$

where  $\tilde{F}(x, h) = F^*(x, -h)$  and  $\sigma_f = 2e^2 N(0) D_f$  is the conductivity of FM-layer. Letting  $I_s = I_c \sin \phi$ , where  $I_c$  is the critical current, we obtain

$$I_c = \frac{\pi T \sigma_f}{2e} \sum_{\omega=-\infty}^{\infty} \frac{8k_1 k_2 \alpha_2 \Delta^2 / (\omega^2 + \Delta^2)}{D}, \quad (12)$$

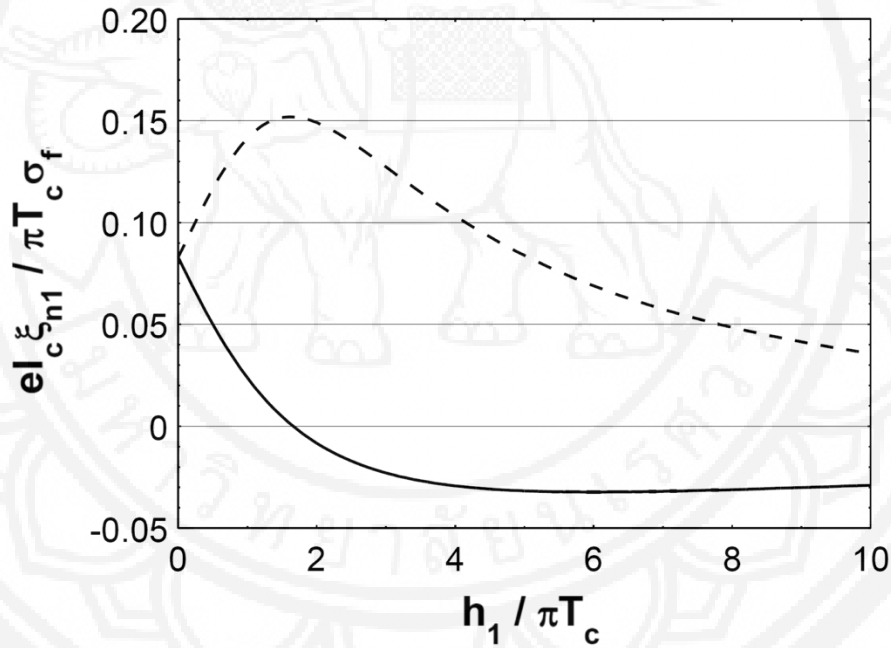
where

$$\begin{aligned}
D = & (|G_s| + \gamma_1 k_1)(|G_s| + \gamma_2 k_2)(\alpha_1 k_1 + \alpha_2 k_2 + \alpha_1 \alpha_2 k_1 k_2) e^{k_1 d_1 + k_2 d_2} \\
& + (|G_s| - \gamma_1 k_1)(|G_s| - \gamma_2 k_2)(-\alpha_1 k_1 - \alpha_2 k_2 + \alpha_1 \alpha_2 k_1 k_2) e^{-(k_1 d_1 + k_2 d_2)} \\
& - (|G_s| + \gamma_1 k_1)(|G_s| - \gamma_2 k_2)(\alpha_1 k_1 - \alpha_2 k_2 - \alpha_1 \alpha_2 k_1 k_2) e^{(k_1 d_1 - k_2 d_2)} \\
& - (|G_s| - \gamma_1 k_1)(|G_s| + \gamma_2 k_2)(-\alpha_1 k_1 + \alpha_2 k_2 - \alpha_1 \alpha_2 k_1 k_2) e^{-(k_1 d_1 - k_2 d_2)}
\end{aligned} \quad (13)$$

The dependences of the critical current on the values of the ferromagnetic exchange fields are through the dependence of the parameter  $k_1$  and  $k_2$  on the exchange fields. For parallel alignment of the magnetizations in the two ferromagnetic layers, the above expression reduces to the critical current of SFS junction given by Buzdin (2003), when the interface barrier is placed at  $x = 0$ ,  $R_0 = 0$ ,  $\alpha_1 = \alpha_2 = 0$ ,  $\gamma_1 = \gamma_2 = \gamma$ ,  $k_1 = k_2 = k$  and  $d_1 = d_2 = d$ .

### Results and discussion

In Figure 2, the critical currents in a SIFM<sub>1</sub>IFM<sub>2</sub>IS junction are plotted as a function of the exchange field in the FM<sub>1</sub>-layer,  $h_1/\pi T_c$ . The value of the exchange field in the FM<sub>2</sub>-layer was set at  $h_2 = 8\pi T_c$ .



**Figure 2** Plot of the critical current of SIFM<sub>1</sub>IFM<sub>2</sub>IS junction (as given by Equation (12)) on the value of the exchange field in the FM<sub>1</sub>-layer. The dashed and solid lines corresponding to the antiparallel and parallel alignments of the magnetizations in the two ferromagnetic layers, respectively. The fixed values of the parameters used in the numerical evaluation of  $I_c(h_1)$  (Equation (12)) are  $T/T_c = 0.2$ ,  $R_1/R_{f1} = R_2/R_{f2} = R_0/R_{f1} = R_0/R_{f2} = 3$ ,  $d_1/\xi_{n1} = 0.4$ ,  $d_2/\xi_{n2} = 0.3$  and  $h_2/\pi T_c = 5$

The thicknesses of the two FM-layers are  $d_1/\xi_{n1} = 0.4$  and  $d_2/\xi_{n2} = 0.3$ . The temperature used to calculate Figure 2 was  $T = 0.5T_c$ . The solid line shows the dependence when the magnetizations in the two FM layers are parallel. As is seen, the critical current decreases as the exchange field is increased. This is expected since

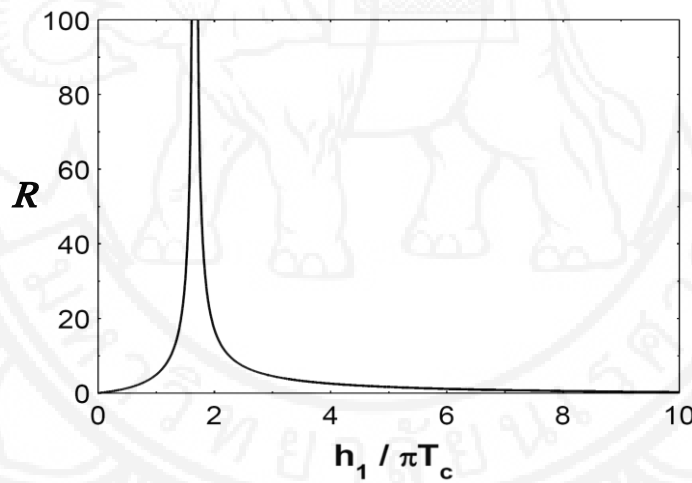


an increase in local magnetic field will suppress the superconducting state. The values of the resistance of the separate layers in the numerical calculations were set at  $R_1/R_{f1} = R_2/R_{f2} = R_0/R_{f1} = R_0/R_{f2} = 3$ . The dashed line shows the dependence of  $I_c(h)$  when the magnetizations in the two layers are antiparallel. It shows that the critical current in a SIFM<sub>1</sub>IFM<sub>2</sub>IS junction where the magnetizations in the two layers are antiparallel is enhanced by the exchange field, a property observed by Bergeret et al. (2001).

To determine under what conditions the maximum enhancement can be achieved, we have calculated the difference between critical currents of the junctions having antiparallel and parallel alignments of the magnetizations in the two layers for different values of  $h_1$  ( $h_2$  was set at  $5\pi T_c$ ). We then calculated a coefficient  $R$  defined as (Bergeret et al., 2005)

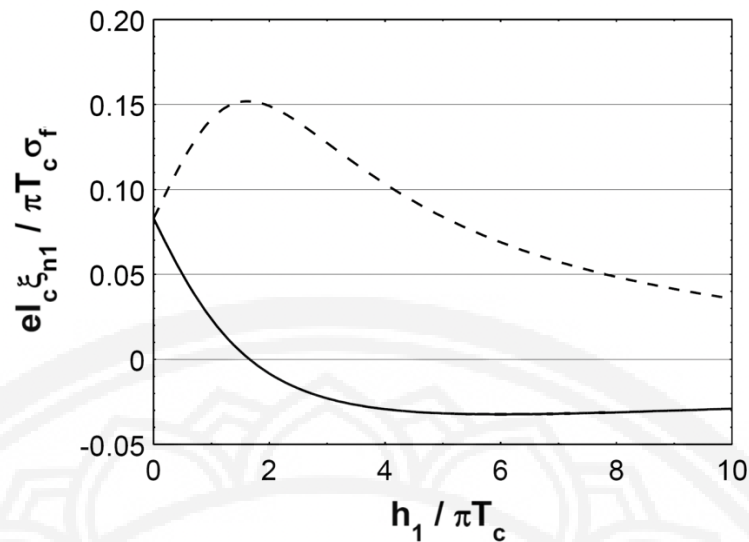
$$R = \frac{|I_c^{(a)}| - |I_c^{(p)}|}{|I_c^{(p)}|}, \quad (14)$$

where  $|I_c^{(a)}|$  and  $|I_c^{(p)}|$  are the absolute values of the critical current for the antiparallel and parallel alignment of the magnetizations in the two ferromagnetic layers. In Figure 3, we have plotted  $R$  as a function of the exchange field  $h_1/\pi T_c$ . Interestingly, the plot exhibits a very sharp peak indicating that the maximum enhancement occurs at some particular value of  $h_1/\pi T_c$ .



**Figure 3** The dependence of the coefficient  $R$  on the value of the exchange field in the FM<sub>1</sub>-layer

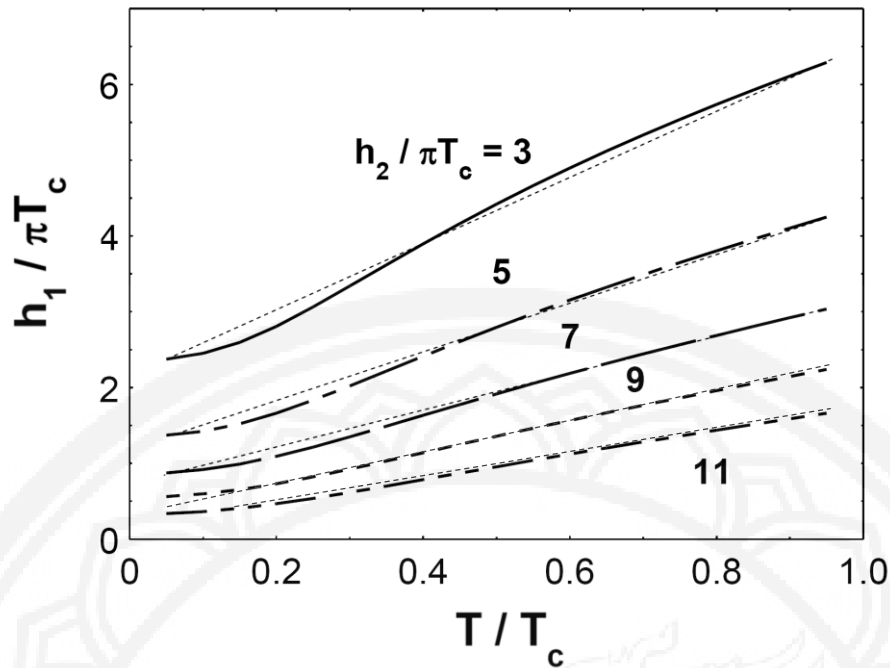
The switching from a 0-state to a  $\pi$ -state junction is seen in Figure 4 where we have plotted the values of the critical current in a junction having parallel alignment of the magnetizations in the two FM layers when the exchange field  $h_1$  is increased.



**Figure 4** Plots of critical currents versus the value of the exchange field in the first ferromagnetic layer  $FM_1$  of a parallel aligned  $SIFM_1IFM_2S$  junction at different temperatures. The cross over point is point where the current changes its direction. These points indicate the switch of a “0 JJ” into a “ $\pi$  JJ”. The temperature at which the evaluations were done are indicated in the insert. The numerical values of the other parameters in Equation (12) used to obtain these graphs are the same as those used to obtain the curve in Figure 2 except for the value of  $T$ . As is clearly indicated, the cross over points occur at higher values of exchange field  $h_1$  when the temperature is raised

When we increased  $h_1$  and set  $h_2$  to a negative number, we did not see any reversal of the critical current. The curves show that the reversal of the critical currents in parallel aligned junctions occurs at higher values of the exchange fields when the temperature is raised. The crossover point is defined as the point where the direction of  $I_c$  goes from being positive to being negative. From trend seen, one can draw the conclusion that the cross over point at which the 0- $\pi$  transition occurs depends on the temperature. We have fixed the values of most parameters and varied the temperatures and the exchange field  $h_2 / \pi T_c$  and obtained a set of figures similar to Figures 2 and 3. From these two sets of figures, we obtained the cross over points for the 0- $\pi$  transition, i.e., the solutions of  $I_c(h_1, h_2, T) = 0$ .

The curves appearing in Figure 5 are the plots of the solutions in the two-dimensional ( $h_1$ - $T$ ) phase space for different values of the second exchange field  $h_2$ . The line indicating the cross over points when the exchange field in the second ferromagnetic layer is very strong ( $h_2 = 11 T_c$ ), is very interesting. It shows that the cross over point can be made to be accessible over the entire temperature range of operation of the SFS junction, i.e.,  $0.05 T_c < T < 0.95 T_c$  by increasing the exchange field in the first layer from  $0.3 \pi T_c$  to  $1.2 \pi T_c$ . In figure 5, the further away  $h_2$  is from  $h_1$ , the stronger the linear relationship of the cross over points for the 0- $\pi$  transition between the  $h_1$  and  $T$  variables. This information may be useful in the design of submicron switches.



**Figure 5** Plot of the solutions of  $I_c(h_1, h_2, T) = 0$  in the two dimensional  $(h_1 - T)$  phase space for different values of  $h_2$ . The solutions of this equation are the points at which the parallel aligned SIFM<sub>1</sub>IFM<sub>2</sub>S junctions undergo the “0- $\pi$ ” transition. The values of the second exchange field are indicated by the label accompanying each curve

### Conclusion

We have presented a general method for calculating the critical current in a SIFM<sub>1</sub>IFM<sub>2</sub>IS junction based on solving the linearized Usadel equation for low transparency interfaces between S and FM in the dirty limit. We have then numerically solve the equation  $I_c(h_1, h_2, T) = 0$ , where  $I_c(h_1, h_2, T)$  is given by Equation (12). The solutions are the points at which the direction of the current are reversed, i.e., the cross over point. The solutions are plotted on the two dimensional  $(h_1 - T)$  phase space in Figure 5. There we find that “0- $\pi$ ” transition can be induced by increasing the strength of the exchange field in the first ferromagnetic layer by a small amount when the exchange field in the second ferromagnetic layer is strong.

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