An Application of the Data Adaptive Linear Decomposition Transform in Transient Detection

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Received 10 April 2003; accepted 21 August 2003

Abstract

The analysis of transient signals is an attractive framework for many problems in digital signal processing. This paper provides an application of the data adaptive linear decomposition transform (LDT) to detect transients in the sinusoidal signal corrupted by an additive impulse noise. The LDT procedure entails obtaining an interpolation error sequence of half-length of the sinusoidal signal, and processing down-sampled data by using a l_p norm optimum interpolation filter. This optimum filter seeks to approximate the odd indexed sequence elements by a linear combination of neighboring even indexed elements to generate the resulting interpolation error sequence. This study shows that a spike being in the interpolation error sequence indicates the location of the transient in the signal, and the effectiveness of the transient detection performance using the l_1-based method compares favorably with the l_2 - and l_∞ -based methods including the popular discrete wavelet transform method.
Keywords: Digital signal processing, Linear decomposition, Adaptive filter, Transient detection.

Introduction

As demonstrated by Cadzow and Yammen (2002), the data adaptive linear decomposition transform plays a very important role in decomposing a signal into an even indexed sequence and an interpolation error sequence. The even indexed sequence is composed of the even indexed elements of the decomposed signal. The resulting interpolation error sequence is obtained from seeking to approximate the odd indexed sequence elements by a linear combination of neighboring even indexed sequence elements via an optimum interpolation filter. It has been shown that both subsequences in each of half-length provide the key for developing such applications as data compression and noise reduction.

In this paper, the interpolation error sequence proposed is slightly different from the interpolation error sequence proposed by Cadzow and Yammen (2002). This error sequence is to play such an important role in detecting transients by using the LDT procedure with the l_1, l_2 and l_∞ norm length-two filters, respectively. This is illustrated by employing the l_1 norm selection in which the performance of such transient signal detection is improved. Its performance is also found to exceed the method of using the one stage of the popular discrete wavelet transform (DWT) with the Daubechies-eight filter.

Basic linear decomposition algorithm

In this section, the linear decomposition transform (LDT) is proposed to describe the way of decomposing a signal in order to generate an interpolation error sequence. Let $\{x(n)\}$ be a finite length real data sequence defined on the interval $0 < n < 2^N$, where N is a positive integer. This data sequence is generated either from a discrete-time process or a continuous-time signal. It is assumed that this data sequence can be written as the sum of two sequences, $\{x(n)\} = \{x_l(n)\} + \{x_s(n)\}$, where $\{x_l(n)\}$ has a long-term behavior and $\{x_s(n)\}$ has a short-term ehavior. Let T be the operator symbol for the linear decomposition transform that maps the data sequence $\{x(n)\}$ into the two half-length sequences $\{c(n)\}$ and $\{d(n)\}$ defined on the interval $0 < n < 2^{N-1}$.

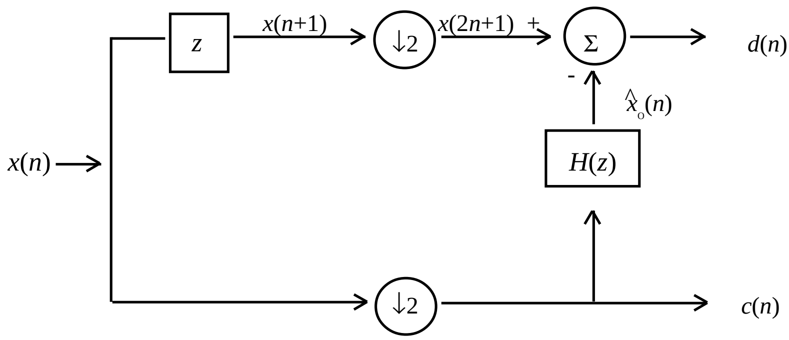

Figure 1. The one-stage of the linear decomposition transform

The one-stage of the linear decomposition transform shown in Figure 1 is defined by

$$(\{c(n)\}, \{d(n)\}) = \mathrm{T}(\{x(n)\}) \quad (1)$$

One half-length sequence $\{c(n)\}$ called an even indexed sequence is composed of the even indexed elements of the data sequence $\{x(n)\}$. Formally, this subsequence is obtained from a down sampling by two operations. The even indexed sequence elements are thus given by

$$c(n) = x(2n) \quad (2)$$

To generate the short-term behavior (i.e., transient) information of the data sequence $\{x(n)\}$, the other half-length sequence {d(n)} called an interpolation error sequence is obtained from the error sequence between the odd indexed sequence $\{x(2n+1)\}$ and its interpolation estimate sequence $\{\hat{x}_o(n)\}$; that is, the elements of the interpolation error sequence $\{d(n)\}$ are given by

$$d(n) = x(2n + 1) - \hat{x}_o(n) \quad (3)$$

Unlike the LDT method proposed by Cadzow and Yammen (2002), the odd indexed sequence element $x(2n+1)$ is obtained by using a standard unit-left-shift operator symbolized by the letter z shown in Figure 1. It is also hypothesized that there exists a meaningful correlation of the odd indexed sequence elements $x(2n+1)$ and its closest $2q$ contiguous even indexed elements $x(2n)$ in which q is a positive integer. The odd indexed interpolation estimate sequence elements therefore is specified by

$$\hat{x}_o(n) = \sum_{k=-q}^{q-1} h(k)x(2n-2k) = \sum_{k=-q}^{q-1} h(k)c(n-k) \tag{4}$$

The $h(k)$ entities are seen to measure the impact that various excitation elements $x(2n)$ have on the present response element estimate $\hat{x}_o(n)$ being computed. The characteristics of the odd indexed estimate sequence are governed by the selection of the $2q$ weighting coefficients $\{h(k)\}$ in relationship (4), so the transfer function of the interpolation filter coefficients $\{h(k)\}$ can be written into the form:

$$H(z) = \sum_{k=-q}^{q-1} h(k)\, z^{-k} \tag{5}$$

It is noted that a linear shift-invariant filter $H(z)$ with infinite length can be obtained by setting $q = \infty$ into relationship (5). Our interest is here concerned with the finite length filter case in which positive integer q is equal to one.

From both relationship (3) and relationship (4), it is obvious that the interpolation error sequence elements defined on the interval $0 \le n < 2^{N-1}$ can be rewritten as

$$d(n) = x(2n+1) - \sum_{k=-q}^{q-1} h(k)x(2n-2k) \tag{6}$$

To achieve satisfactory transient detection performance, the $\{h(k)\}$ interpolation filter's weighting coefficients are selected to cause the $\{d(n)\}$ interpolation error sequence to have as many small magnitude components as possible. This selection process is made to have the interpolation error sequence contained the transient information of the data sequence being decomposed. While selecting the interpolation filter coefficients in relationship (6), the steady state behavior of the $\{d(n)\}$ elements can be written as the convenient vector format:

$$\bar{\mathbf{d}} = \bar{\mathbf{y}} - A\bar{\mathbf{h}}\,, \tag{7}$$

where the $(2^{N-1} - 2q + 1)$x1 error vector $\bar{\mathbf{d}}$ is a given set of the steady state interpolation error sequence elements $\{d(q-1), d(q), \ldots, d(2^{N-1}-q-1)\}$, the $(2^{N-1}-2q+1)$x1 data vector $\bar{\mathbf{y}}$ is a given set of the odd indexed sequence elements $\{x(2q-1), x(2q+1), \ldots, x(2^{N}-2q-1)\}$, the $(2^{N-1}-2q+1)$x$2q$ data matrix A is a rectangular arrangement of a given set of scalar elements $a(m,n) = x(4q-2(n-m+1))$ such that m and n are defined on the positive integer intervals $1 \le m \le 2^{N-1} - 2q-1$ and $1 \le n \le 2q$, respectively.

The $2q$x1 parameter vector $\bar{\mathbf{h}}$ appearing in relationship (7) is a set of interpolation filter coefficients $\{h(-q), h(-q+1),\ldots, h(q-1)\}$. Again, to achieve a good transient detection performance, our objective is to select an optimum parameter vector $\bar{\mathbf{h}}^o$ in order to cause the error vector $\bar{\mathbf{d}}$ to have the minimum l_P norm induced functional given by:

$$\min_{\bar{\mathbf{h}} \in R^{2qx1}} \left\| \bar{\mathbf{y}} - A\bar{\mathbf{h}} \right\|_P = \left\| \bar{\mathbf{y}} - A\bar{\mathbf{h}}^o \right\|_P \quad (8)$$

Our primary interest to select these interpolation filter coefficients is confined to the norm cases: $p = 1$, $p = 2$, and $p = \infty$ (Cadzow, 2002). The development of the l_2 norm approximate solution $\bar{\mathbf{h}}^o$ is well known and found in many references (Cadzow, 2002). A necessary condition is that this l_2 norm approximate solution $\bar{\mathbf{h}}^o$ is equal to one, which satisfies the so-called *normal system equations* (Luenberger, 1969) specified by $(A^T A)\bar{\mathbf{h}}^o = A^T \bar{\mathbf{y}}$. Therefore, if the matrix A is invertible, then the optimum l_2 norm selection filter coefficient vector is given by

$$\bar{\mathbf{h}}^o = (A^T A)^{-1} A^T \bar{\mathbf{y}} \quad (9)$$

If the matrix A is not the case, then $\bar{\mathbf{h}}^o = A^T \bar{\mathbf{y}}$, where A^T designates the pseudo inverse of the matrix A. For the rest two cases: $p = 1$ and $p = \infty$, a closed form relationship for either the minimum l_1-norm approximate solution or the minimum l_∞-norm approximate solution does not exist simply. One must therefore resort to algorithmic procedures (i.e., nonlinear programming methods) for iteratively finding their determination. In this paper, the optimum either l_1 or l_∞-norm selection filter coefficient vector $\bar{\mathbf{h}}^o$ is computed by using the effective algorithms developed by Cadzow (2002).

To illustrate the capability of the proposed data adaptive linear decomposition transform on transient signal detection, its performance on a set of studied one-dimensional sinusoidal signal corrupted with three impulse discontinuities is given. We then examine the performance of the basic data adaptive linear decomposition transform on the test signal to detect the three transients. In this experiment, these results obtained from employing the l_1, l_2 , and l_∞-norm optimum length-two interpolation filters are also compared with the results obtained from using the Daubechies-eight filter of the traditional wavelet transform (Chui, 1992; Daubechies, 1992).

Transient detection results

In order to evaluate the transient detection performance of the data adaptive linear decomposition transform on a one-dimensional sinusoidal data signal corrupted with three impulse discontinuities, the studied signal is shown in Figure 2, and takes the form:

$$x(n) = 15\sin(n\pi/16) + 0.4\delta(n-35) + 0.4\delta(n-67) + \delta(n-103), \quad (10)$$

where n is an integer index defined over the intervals $0 \le n \le 127$. This test signal has three discontinuities at the points n = 35, 67, and 103. Employing the one stage of the *Daubechies-eight discrete wavelet transform* (Flandrin, 1999; Mallat, 1998) and the one stage of the data adaptive linear decomposition transform with three length-two interpolation filters ($q = 1$) on the test signal, the transient detection results shown in Figure 3 and 4 were obtained. The three length-two interpolation filters used in the one stage of the data adaptive linear decomposition transform were obtained by minimizing the l_1, l_2 and l_∞-norm of the interpolation sequences. Figure 3 shows that the three transients were captured in the wavelet detail sequence using the *Daubechies-8 filter* (a filter of length-eight) (Rao and Boparadikar, 1998).

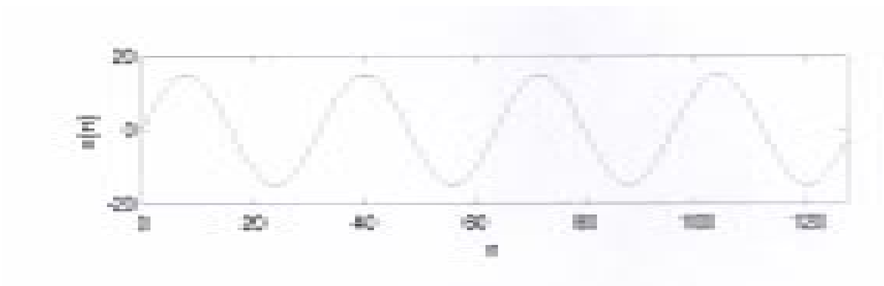

Figure 2. The studied signal

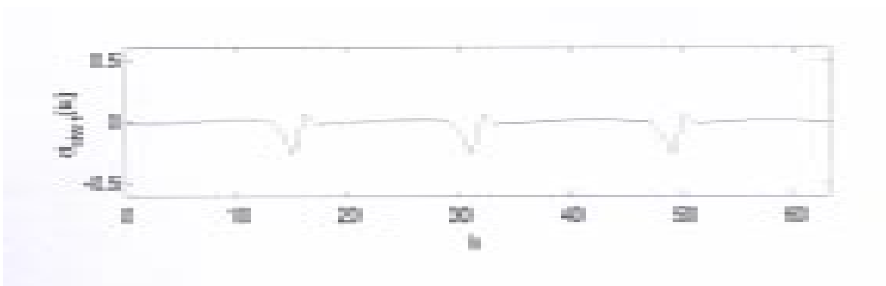

Figure 3. Transient detection using the daubechies-8 DWT procedure for the studied signal

The three transients shown in Figure 4 were also detected in the $\{d(k)\}$ interpolation error sequence using the optimum length-two interpolation filters by minimizing l_1, l_2 and l_∞ -norm of the interpolation error sequence, respectively. It is noted that each discontinuity captured by using the DWT procedure has two positions while the one detected by using the LDT procedure has exactly one position. With these results, the l_1 norm leads to superior performance of the transient detection relative to the DWT, the l_2, the l_∞-based methods.

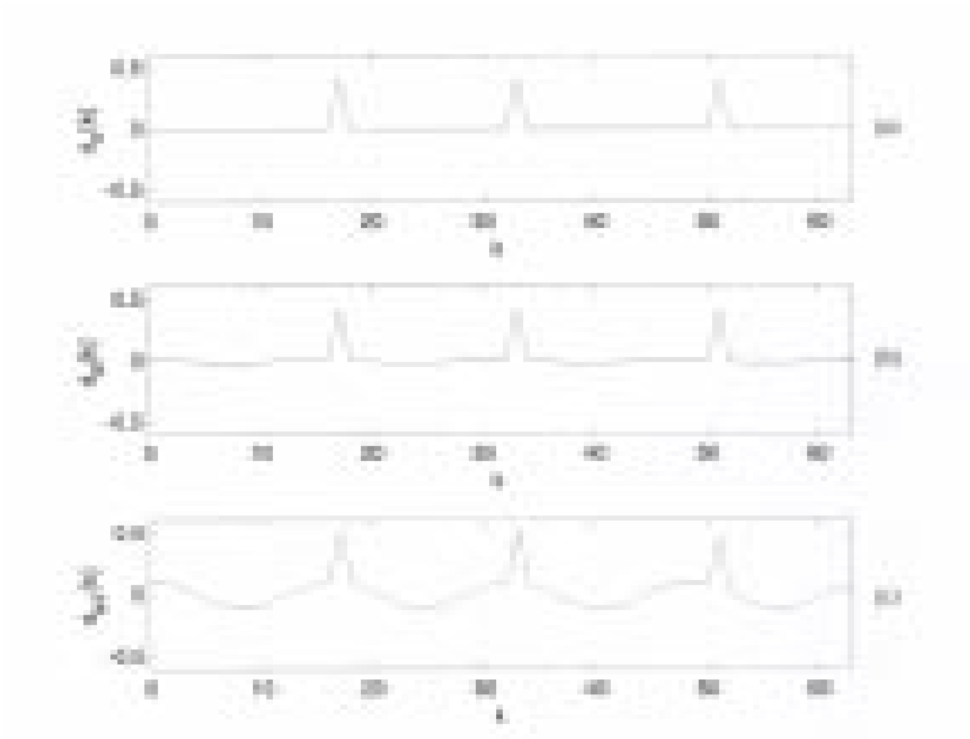

Figure 4. Transient detection using the LDT procedure for the studied signal: (a) LDT using l_1 norm, (b) LDT using l_2 norm, (c) LDT using l_∞ norm

Discussions and conclusions

Even though we cannot see the three discontinuities from the noise corrupted sinusoidal signal shown in Figure 2, these transients were detected by the *Daubechies-8* DWT detail sequence using a filter of length eight and LDT interpolation error sequences using length-two filters in the one stage of their decomposition procedures. In our results, the different length of the filters was employed because the l_1 and l_2 optimum length-two interpolation filters can exactly represent a pure sinusoidal signal while the *Daubechies-8* wavelet with longer supports of length-eight was used to exactly represent this pure sinusoidal signal.

In this paper, we have presented a LDT method for adaptively decomposing one-dimensional interpolation error sequence into one-half length of a transient signal. Simulation results have shown that this interpolation error sequence obtained from selecting the optimum l_p norm interpolation filter can be used to detect discontinuities in the transient sinusoidal signal. Furthermore, its effectiveness of the transient detection performance using the l_1-based method compares favorably with the l_2-, the l_∞ -, and the DWT-based methods.

Acknowledgements

The author is grateful to Krisanee Yammen, Dr. Paisarn Muneesawang, Siriporn Dachasilaruk and Panote Wilaipon for their very helpful suggestions during either writing or revising stage in this paper. The author also thanks to Mr. Kunihiko Ikuzaki for valuable comments and discussions concerning his paper.

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